TUTORIAL # 3

"CONCEPTS MODERNES, OBJECTIFS ET PROJETS SATELLITAIRES POUR LA DÉTERMINATION ET L'UTILISATION DU CHAMP DE GRAVITÉ TERRESTRE"

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Résumé

Ce cours, pour le moment en Anglais, est une adaptation tirée de la publication ESA N-SP-1196 (1) : "Gravity Field and Steady-State Ocean Circulation Mission" (GOCE), écrite en 1996 par les quatre mêmes auteurs. Il s'attache à décrire précisément les concepts liés au champ de gravité de la Terre dont ont besoin la géodésie, la géophysique et l'océanographie pour un large éventail d'études scientifiques et d'applications. Dans chaque domaine, on rappelle l'état de l'art et l'on détaille ce qui est nécessaire pour progresser et qui passe par l'amélioration obligatoire de notre connaissance du champ. On donne les principes de systèmes spatiaux actuellement à l'étude pour déterminer un modèle de champ de plus grande résolution, et plus précis. L'un de ces projets de mission, GOCE, utilise simultanément les deux techniques de poursuite de satellite par satellite (et la constellation GPS) et de gradiométrie embarquée. Des résultats de simulation sont donnés qui démontrent l'impact très important qu'aurait une telle mission dans les trois disciplines concernées.

"MODERN CONCEPTS, CONCERNS AND SATELLITE PROJECTS IN THE DETERMINATION AND USE OF THE EARTH'S GRAVITY FIELD"

Foreword

This text is adapted from the first chapters of the ESA publication N. SP-1196 (1), entitled : "Gravity Field and Steady-State Ocean Circulation Mission" (GOCE), written by the same authors in 1996. It was a Report for Assessment addressing the GOCE mission which aim is to provide global and regional models of the Earth's gravity field and of the geoid, its equipotential surface, with high spatial resolution and accuracy. Such models would be used in a wide range of research and application areas, including global ocean circulation, physics of the Earth's interior and levelling systems based on a Global Positioning System (GPS).

We found these chapters sufficiently general and self-contained for being included in the BGI series of tutorials.

1. INTRODUCTION

The determination of the gravitational potential of the Earth is a fundamental task of geodesy which aims at improving the knowledge of the shape of our planet, of position of points on its surface for various applications (cartography, hydrography, road construction ...), but it is also of extreme importance in geophysics since it provides key information which, combined with other data - topographic, seismic ..., constitute essential constraints on models of the interior. Besides, modern oceanography requires a precise reference surface (the geoid) to improve its understanding of the circulation of the oceans.

Although there has been a breakthrough in geopotential modeling in the recent years thanks to the improvement of satellite tracking systems (laser, Doris, GPS) and the analysis of orbital perturbations on one hand, and thanks to the advent of satellite altimetry on the other hand, the former yielding a better knowledge of the long wavelength variations and the latter bringing much higher resolution in a surface close to the geoid over oceanic parts, it remains that a great deal of efforts have still to be done to improve the global accuracy and resolution of Earth's gravity field models.

Ground techniques and conventional satellite methods cannot be used much further for global investigations at higher resolution, and new ways of determining gravity variations in the resolution range of 50 to 200 km, especially over lands, have to be explored.
It is the aim of this tutorial to show what are the present concerns about the mapping and utilization of the Earth's gravitational potential for geodesy, geophysics of the solid Earth and oceanography in an area where scientific curiosity and needed applications merge, that is for a better knowledge of our environment and more precisely for improving our capacity at predicting the evolution of the Earth's climate through the ocean circulation, its level changes, the behaviour of the ice caps, etc.

2. BACKGROUND AND SCIENTIFIC JUSTIFICATION

2.1. Basic Concepts

We summarise below the main facts on the physics of the Earth in the domain of geodesy, geodynamics and oceanography, which are necessary to understand the arguments which will be further developed. A more detailed and historical account can be found in tutorials # 2 and # 4.

2.1.1. Basic gravity field quantities. Definition and methods of observation

Figure 2.1. shows the main quantities we will be dealing with.

The gravity potential, *W*, is the sum of the potential generated by the attraction of masses *(V)* and the centrifugal potential. Differences between values in two points may be observed by levelling. A surface where *W* is equal to a constant is an equipotential surface. Points on one such surface may be determined regionally by tide-gauges, which define regional mean sea level. A height datum is defined by the equipotential surface which best agrees with *local* mean sea level calculated from tide-gauges for a specific time period. The equipotential surface which approximates the *global* mean sea level, i.e. a global set of tide-gauges and levelling bench-marks, after subtraction of the dynamic components, is the geoid.

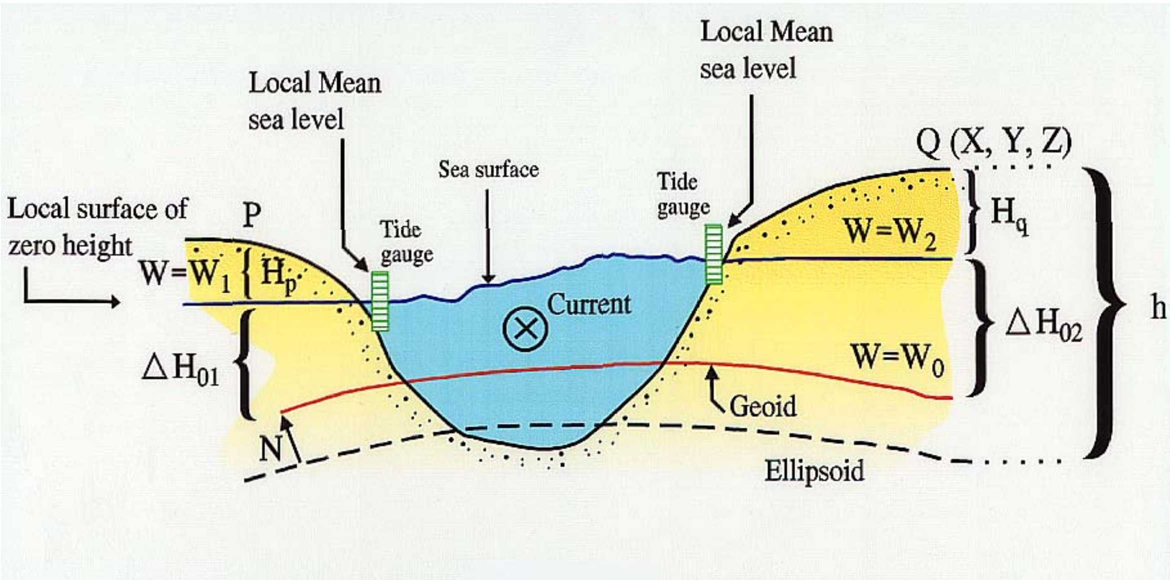


Figure 2.1. *The main geodetic quantities used in this tutorial, and illustration of a basic problem: Order of magnitude of* $\Delta H_0 = \Delta H_{02} - \Delta H_{01}$ *is 1 m between continents, and 0.1 - 0.2 m between islands and from coasts of continents to islands. Other symbols are defined in the text.*

The mean Earth ellipsoid is an ellipsoid of revolution, rotating with the Earth around its Z-axis, and centered at the Earth's centre of mass. It is determined as the surface which gives best fit in some sense to mean sea-level. The height above this ellipsoid, *h*, is measured along the normal to the ellipsoid. It is observed indirectly by satellite positioning from the determined cartesian coordinates (*X,Y,Z*). The geoid height, *N*, is the height of a point on the geoid above the ellipsoid. It can be observed by determining *h* by satellite technique at a tide-gauge or at a levelling point. The orthometric height, *H*, is measured from the geoid along the plumbline (it is often called the height above mean sea-level). It is observed by levelling: the measurements (level differences and gravity) yield the geopotential number which is converted to metric units by dividing by the mean gravity along the plumbline.

Gravity, g, is the magnitude of the gradient of W at the Earth's surface and of V in space. It may be observed by absolute technique (e.g. in a free fall experiment) or relatively (as a difference) by a spring gravimeter. Gravity gradients are derivatives of the gravity vector, i.e. second order derivatives of W. Certain linear combinations may be measured by a torsion-balance at Earth's surface, and by forming differences of accelerometer measurements in space.

A model (normal) gravity potential, U, with the ellipsoid as an equipotential surface, is used to calculate normal gravity, γ. At any point of given latitude and orthometric height, the gravity anomaly Δg is the value derived by subtracting measured and normal gravity $(\Delta g = g - \gamma)$. The gravity γ is calculated at a point with the ellipsoidal height put equal to the orthometric height. This is the modern definition of Δg according to Molodensky, which differs from the gravity anomaly at sea level which is still used sometimes (see Heiskanen and Moritz, 1967). The difference $T = W - U$ is called the anomalous potential. It is small and allows linearization, such as the Bruns equation $N = T/\gamma$, which directly relates potential and geoid height. Measured quantities are frequently expressed as derivatives of T, such as the gravity gradients.

The potential V (or T) may be expanded as a series of spherical harmonic functions which are the spherical equivalent of Fourier series in a plane. The coefficients of the series are numbered according to degree and order, l and m respectively ($m \leq l$), which correspond to wave-numbers in the plane. The zonal harmonics are those coefficients of order zero and correspond to averages of the potential in longitude. The other coefficients are called tesseral harmonics (sectorial when $l = m$). For a given degree, the quadratic mean (over all orders) of the harmonic coefficients for the Earth approximately decrease like $10^{-5}/l^2$ (Kaula's rule).

A global model of the Earth's gravitational potential is a set of harmonic coefficients, truncated at a maximum degree and order L, which corresponds to a spatial resolution (maximum resolved half-wavelength) of 20 000 km/L.

Units and orders of magnitude are the following: the gravity is expressed in m/s^2 or in milligal (1 mgal = 10^{-5} m/s^2); the mean Earth gravity is about 981 000 mgal, varies from 978 100 mgal to 983 200 mgal from Equator to pole due to the Earth's flattening and rotation. Excursions due to density homogeneities, mountain ridges, etc ... range from tens to hundreds of milligals. On the other hand, the excursions of the geoid, measured from the mean Earth ellipsoid, amount to about + 90 and - 105 meters. Gravity gradients are expressed in Eötvös (1 E = $10^{-9}s^{-2}$). The largest component is the vertical gravity gradient, being on Earth about 3000 E (gravity changes by 3.10^{-6} m/s^2 per metre of elevation). The horizontal components are approximately half this size, mixed gradients are below 100 E for the normal field. Gravity gradient anomalies can be much larger and reach 1000 E in mountainous areas.

2.1.2. Geodynamics

In a section through the centre of the Earth, Fig. 2.2. portrays a simplified picture of the interior, inferred from geophysical studies; this picture helps us to introduce the terminology that will be used in the following sections devoted to solid Earth geophysics to indicate the various layers in which the planet is differentiated and the major geodynamical processes that will be discussed and whose physical understanding will gain from a high resolution gravity mission. The mechanically stiff outer layer is called the lithosphere, which is, in turn, subdivided into an oceanic and a continental part. The major geodynamical processes that involve the oceanic lithosphere are spreading at the ocean ridges, and subduction as indicated in the picture. The cold, dense oceanic lithosphere enters the mantle at subduction zones, interacting with the overriding lithosphere, where complicated geodynamical processes, such as back arc opening and volcanism occur. The arrow at the subduction zone indicates the velocity of the plate with respect to the mantle, controlled by the downward pull exerted by the cold subducted plate, the push at the ocean ridge and the basal viscous drag.

One of the major issues concerning the dynamics of ocean plates is the structure and evolution of the region where an ocean plate bends to initiate subduction at an ocean trench. Opening of back arc basin, volcanism, represented by the volcanic line in the region called back arc in the figure, rates of subsidence in the trench, indicated by the arrow, are in fact ultimately controlled by the dynamic interaction between the ocean and overriding plates. Ocean ridges are the locations where the oceanic lithosphere is generated by means of spreading, as shown by the horizontal arrows; subduction and spreading are major processes of plate tectonics.

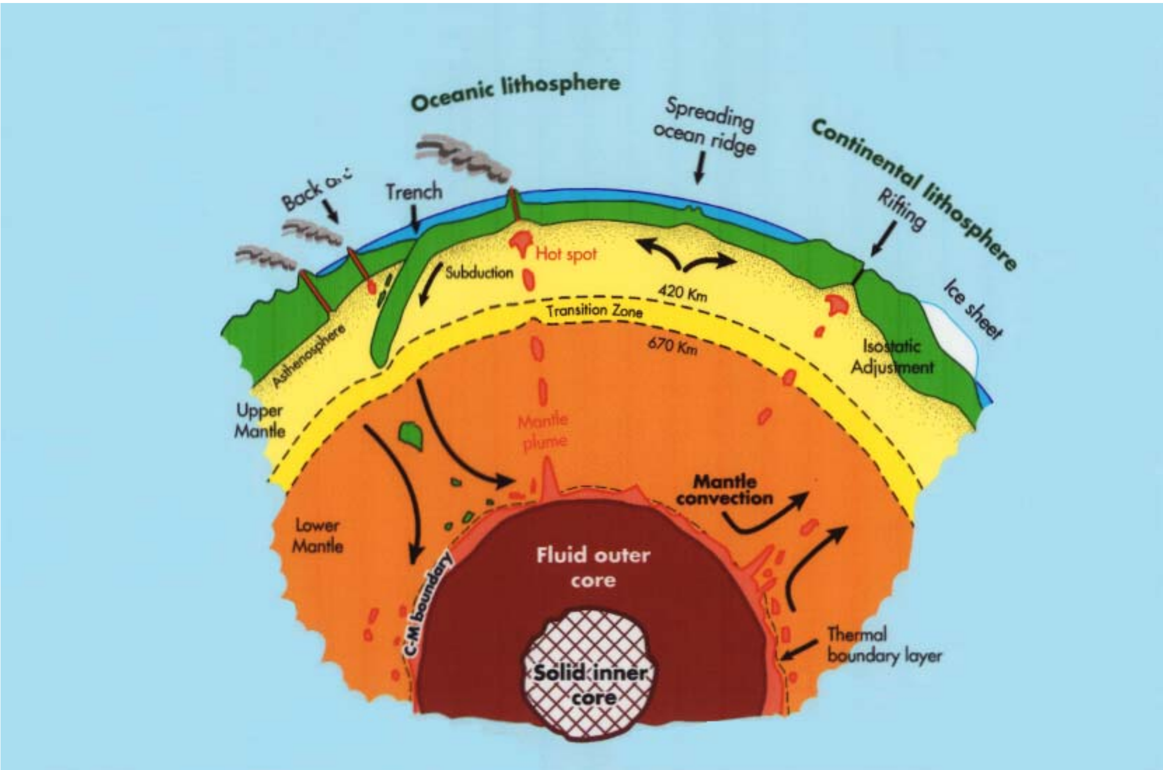


Figure 2.2. *The major dynamic processes of plate tectonics and the layers in which the Earth is differentiated. From the outer lithosphere, divided into the ocean and continental parts, we recognize the upper mantle, the transition zone and the lower mantle. The fluid outer core and solid inner core are also portrayed. From left to right, we recognise subduction and related back*

arc opening, hot spots and a spreading ocean ridge. For the continental lithosphere the isostatic adjustment following post-glacial rebound is suggested.

It is important to improve our knowledge of the structure of the continental lithosphere, because it is the location of strategic mineral resources. The continental lithosphere has been the place of the continuous occurrence of glaciation and deglaciation events, at least in the last million years. The last deglaciation ended about 7000 years ago, and the planet is still recovering its condition of isostatic equilibrium after the unloading of the lithosphere due to the melting of the ice-sheets. The response of the planet to these events and the associated gravity anomalies depend on mantle rheology which tells us how mantle material responds to deviatoric stresses. This process is called post-glacial rebound (PGR), which will be discussed in the following together with the potential improvement in its understanding thanks to a high resolution gravity mission. A major issue related to the structure of the continental lithosphere is the possible existence of deep roots beneath the continents. Deformation of the continental lithosphere under the influence of extensional forces is visible as elongated depressions called rifts, that are present in a variety of tectonic environments.

Studies on rheology are also important for our comprehension of other mechanisms that involve the mantle, the portion of the planet beneath the lithosphere down to the core mantle boundary; mantle convection, depicted by the arrows is certainly one of these, involving the circulation of mantle material on geologic time-scales. Relevant for these studies is also the dynamics of fast upwelling plumes in the mantle that are responsible for the appearance of hot spots in the lithosphere.
The mantle is subdivided into an upper and a lower mantle by a transition zone, between 420 and 670 km, depicted in the figure by the dotted strip. It will be shown that a high resolution gravity mission will help constrain the structure and the nature of this transition zone that is expected to play a major role on the convective style within the mantle. The low viscosity region beneath the lithosphere, and the uppermost part of the upper mantle is called asthenosphere, represented by the shaded area above the transition zone (fig. 2.2.).

2.1.3. Ocean Circulation

Oceanographers want to know the global 3-dimensional pattern of ocean currents induced by meteorological and thermo-haline forcings. In order to do this, one could imagine an ocean covered with thousands of current meters, floats and drifters, and in fact modest numbers of such devices are at present being deployed as part of WOCE. However, truly-global coverage by this means would be impossible, and would inevitably be inhomogeneous. This is all the more difficult as the ocean circulation is constantly changing on many time scales. Fortunately, one can infer the ocean circulation from its 'dynamic topography' signal which is the difference between a mean sea surface, as measured by precise satellite radar altimetry, and the geoid. A large part of the surface circulation is in geostrophic balance with the ocean's surface pressure gradient (or dynamic topography gradient) in an analogous fashion to the way winds are balanced by air pressure gradients in the atmosphere.

Figure 2.3. shows an estimate of the mean (i.e. time averaged) dynamic topography from a numerical ocean model. Amplitudes of approximately ± 1 metre can be clearly identified in the areas of the major currents (Gulf Stream, Kuroshio, Antarctic Circumpolar Current etc.), and it is this signal which we want to measure in reality on a global basis and to high spatial resolution. This signal can be seen to be two orders of magnitude smaller than that of the geoid itself (Figure 2.4.).

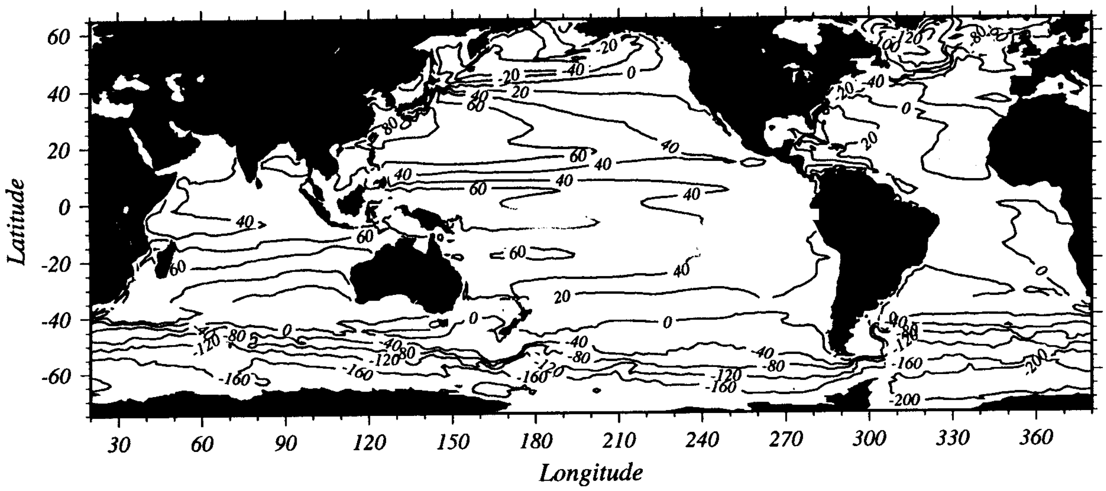


Figure 2.3. *Mean sea surface dynamic topography from an 8 year run of the Semtner-Chervin numerical ocean model. Contour interval is 20 cm. N of 40°S and 40 cm, below. (Courtesy Dr. Robin Tokmakian).*

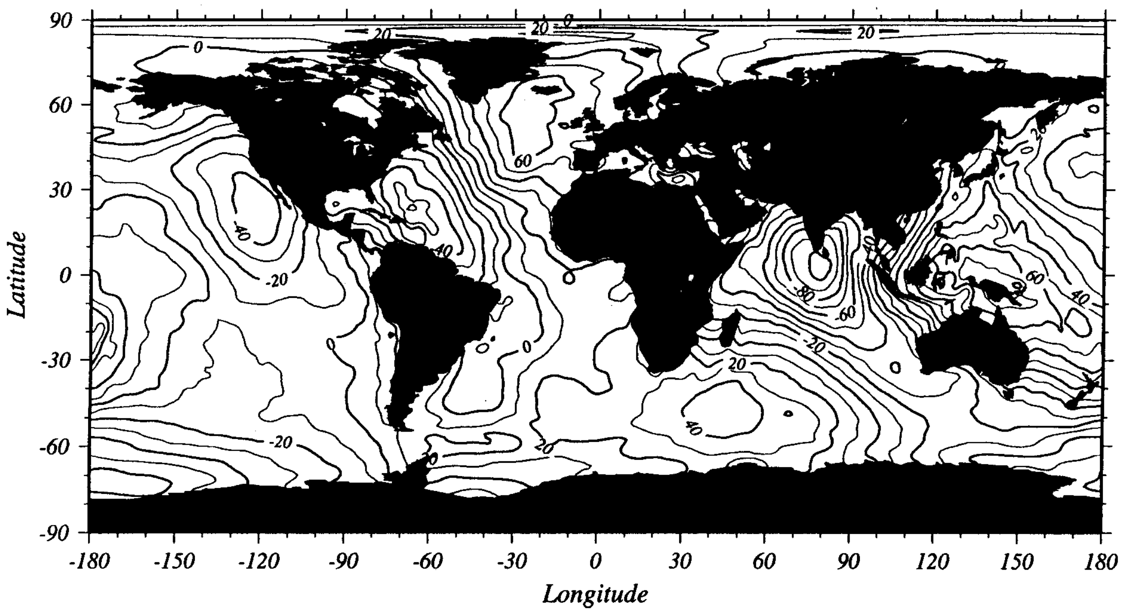


Figure 2.4. *The geoid heights of the GRIM4-C4 model with respect to the ellipsoid best approximating the Earth. Contour intervals is 10 m.*

The ocean circulation may be further subdivided into 'barotropic' and 'baroclinic' flows. Ideal barotropic conditions apply where surfaces of constant density throughout the water column (isopycnals) are parallel to the surfaces of constant pressure (isobars) owing to there being little or no lateral variation in density. The dynamic topography (or sea level) gradient, which is the surface horizontal pressure gradient, determines not only the surface current through geostrophy, but also ensures that the same current pertains at all depths as the horizontal pressure gradient is the same at each depth; i.e. there is no vertical velocity shear. Hydrographic (density) measurements from a ship *can not* be used to determine barotropic flow from the density distribution as in this case the same density profile is obtained at each location; only knowledge of the surface pressure gradient can give us the information we need. Idealised baroclinic conditions apply where there are strong lateral variations in density, and the isobars at depth are not parallel either to the isopycnals or to the surface horizontal pressure gradient. Consequently, there are velocity shears between different depths. Large parts of the ocean circulation have flows determined by baroclinic conditions, which historically have enabled oceanographers to use hydrographic measurements throughout the ocean to determine the shape of the isopycnic surfaces and, together with an assumption of a 'level of no motion' at great depth (typically at 2000 m), to infer a first order approximation of the surface circulation. However, two centuries of hydrography have not told us all we need to know about even the baroclinic component of the circulation: horizontal resolutions of hydrographic measurements are generally poor compared to altimetry; some regions (e.g. S.E. Pacific) have virtually no hydrographic data; and, since each set of measurements was made at different times, questions of temporal aliasing arise.

To summarise, in common oceanographic parlance, 'baroclinic' refers to the vertical shear of currents, calculable from hydrography, and 'barotropic' refers to the vertically averaged current (or sometimes the current at some reference depth). As explained above, whether flows are primarily barotropic or baroclinic, the overall geostrophic surface currents are determined to a good approximation from the sea level (or dynamic topography) gradient, which is what precise altimetry and a precise global gravitational model will together provide. (Ekman wind-driven transports are also important at the surface but they have no sea level gradient signature and are not in geostrophic balance. They are easily calculable from the wind field). Subsequently, determination of the total volume and heat transports, by assimilation of the surface current information with WOCE-provided hydrographic information at depth into numerical models, will provide a complete picture of the 3-dimensional current and thermal structure of the ocean. That will be a large computing task but oceanographers believe that they already possess the necessary techniques.

To give a simple example, imagine a sea level difference of 10 cm across a zonal section of ocean 1000 km wide at 45 deg N. From geostrophy, we can compute the consequent meridional surface current of velocity v_y :

$$v_y = g/f \, . \, dh/dx$$

where g = 1000 cm/sec^2 is the approximate acceleration due to gravity

f = 10^{-4} sec^{-1} is the approximate Coriolis parameter at 45 deg N
and dh/dx = 10 cm / 1000 km is the sea surface gradient.

In this case the meridional current is approximately 1 cm/sec. Now, if the ocean section is 1000 m deep and barotropic conditions apply, then the meridional volume transport is 10^7 m^3/sec or 10 Sv, where 1 Sverdrup (Sv) = 10^6 m^3/sec. If the flow is between two basins, the first 1°C warmer than the other, then the volume transport may be multiplied by the specific heat of water (approx. 4.2 J per gm per deg C) to give a corresponding heat transport of 4.10^{13} W.

2.2. Studying the Gravity Field

More and more emphasis is being given to the study of the Earth as one system. Fundamental questions concerning the exact nature of the dynamics of the solid Earth and oceans and the interaction of the latter with the atmosphere are unsettled. In this context, global modelling of the Earth’s gravitational potential is still one of the basic areas of research in geophysics, since the gravity field, in addition to the analysis of seismic wave propagation and of the magnetic field, is the most valuable source of information about the nature and composition of our planet, and evolutionary processes which continue to shape it. It is also with respect to one preferred equipotential surface, the geoid, that the conventional altitudes can be determined and that the ocean circulation can be best described and studied. Space measurements of gravity are required since ground observations are hard to make world-wide especially over the oceans and in areas of difficult access. The oldest approach, which derives the spherical harmonics coefficients describing the gravity potential from the analysis of artificial satellite orbit perturbations, is limited: a single satellite orbit is mostly sensitive to certain classes of spherical harmonics and therefore a great variety of satellites is needed; then satellites have to fly higher than a minimum altitude (which in the past has been limited to ~ 500 km to ensure a sufficient lifetime) as dictated by the upper atmosphere. Consequently, since the orbital perturbations in the average decrease quasi exponentially with the wave-number, the maximum resolution of a global model obtained in this way is quite limited: it is approximately equal to the satellite flight altitude. Present models (Nerem et al., 1995; Schwintzer et al., 1996) have naturally taken into account the ground gravity data, as well as the information derived from satellite altimetry (which provides the sum of the geoid height - with respect to an ellipsoid, and of the height of the ocean surface with respect to the geoid). However, these solutions barely reach a resolution of 400 km (half wavelength) with larger errors below 2000 km. Naturally where dense and good quality surface gravity data exist, the geoid and the gravity field are much better known, but such favourable cases are limited to a small number of land areas.

Why is it necessary to go further ?

In the last fifteen years or so geophysicists have had a wealth of information on the marine gravity field as deduced from satellite altimetry (provided that the sea surface topography correction is deemed satisfactory at the studied wavelengths) and this was indeed a revolution in the Earth Sciences from which new concepts and new models arose. Great questions now are posed about the continental areas: here the lithosphere is certainly of very different nature since it supports the continents. Upcoming airborne techniques, which start to benefit from improved instrumentation and more accurate positioning thanks to GPS, are improving but are uneconomic on a global scale. It would be awfully expensive if all poorly known continental areas had to be covered that way.

The other main reason for improving our knowledge of the geopotential resides in the relationship between the ocean circulation and global climate. The circulation is a fundamental process for sustaining life on Earth as, on average, it transports as much heat as the atmosphere does from the Equator to the poles, and will play a central role should climate change to any extent owing to 'enhanced greenhouse forcing'. Oceanographers need to be able to measure the mean ocean circulation, by almost two orders of magnitude more accurately than they can to-day. At the present time this calculation is dominated by errors in geoid models, hence the great oceanographic interest in a mission to improve them. Present decimetric geoid model errors in the areas of the major ocean currents result in errors of order 10-20 percent of their typically 100 Sv volume transports.

Gravity or geoid signatures can also be used for early detection of global changes and, equally important, to understand their underlying mechanisms. Many evolutionary processes influencing global change express themselves either directly or indirectly through changes in the gravity signal. For example, gravity signals are expected from accumulations or reductions in glacier mass, which accompany sea level changes. Unfortunately, the present knowledge of the gravity field is insufficient to allow prediction of global trends from short term measurements of either local or global gravity signals. High resolution global gravity measurements are therefore needed to establish a baseline for global change studies.

Hence improvement of our current knowledge of the global gravitational potential is undoubtedly an item of highest priority. The approach from space is the only one capable of providing global and homogeneous information within a reasonable time frame. It must be made, almost independently from the techniques to be used, from an orbit as low as possible for maximum gravity signal, with almost polar inclination for global world-wide coverage, and quasi-circular for homogeneous sensitivity. A space mission fully dedicated to the gravity field would be the first of its kind and would represent an extraordinary step forward in the understanding of the planet on which we live.

2.3. Current status of gravity field knowledge: global, regional, local

At present we have several sources of gravity field information, which we here may divide in two groups: directly observed quantities and indirectly observed quantities.

2.3.1. Directly observed quantities

Potential differences

Differences of values of the gravity potential W are observed by levelling combined with gravimetry. The coastal regions of the main continents and islands are covered with levelling networks (bench marks) which are used both for engineering purposes as well as for scientific purposes. The potential differences are used to compute orthometric heights (H). Due to the costly observational procedures this type of data are primarily collected along main roads and at low altitudes. When the levelling measurements are connected to (one or more) tide gauges, which define a local height datum, the heights are all given in this datum (see fig. 2.1.). The heights in the local datum will differ from the height in a global datum by a common value ΔH_o, of the order of 0.1 - 0.5 m.

Geoid heights or height anomalies

If the ellipsoidal height h is measured in a levelling point (a bench mark), differences between geoid heights are obtained: $\delta N = N(P) - N(Q) = h(P) - H(P) - (h(Q) - H(Q))$. Only if we have obtained a connection to a global height datum (ΔH_o) is it possible to obtain "absolute" geoid heights. The height of the sea-surface is determined by satellite radar altimetry. In areas with small currents these heights may be regarded as geoid heights and processed in order to obtain gravity anomalies (see below) or components of the gravity vector direction. In this way the oceans have been covered with gravity anomaly estimates with a spatial resolution of 10 km at mid latitudes. In areas with significant ocean currents these values will have large errors (see section 2.4.3).

Gravity

Absolute values of gravity are available at a few hundred points. These points are used as the basis for measuring values at the solid earth surface with gravimeters which precisely measure the difference between points. Besides measuring the gravity or gravity differences the height must also be precisely known. Currently 2.5 million values measured on land are available for scientific use. Gravimeters are used at the earth's surface and, when properly damped, in ships and aircraft. The measurements on ships can be regarded as along-track filtered values of a wavelength of about 1 km, while the values observed on aircraft are along track values of about 15 km wavelength. Currently 10 million values are measured at sea and 100000 values by airborne observations. Fig. 2.5. depicts the distribution of presently available measurements (over lands and oceans) in the data base of the Bureau Gravimétrique International.

The gravity values are converted to so-called gravity anomalies by subtraction of a reference gravity value calculated from the knowledge of the latitude of the observation point and of the orthometric height of the point (cf. 2.1.1.).

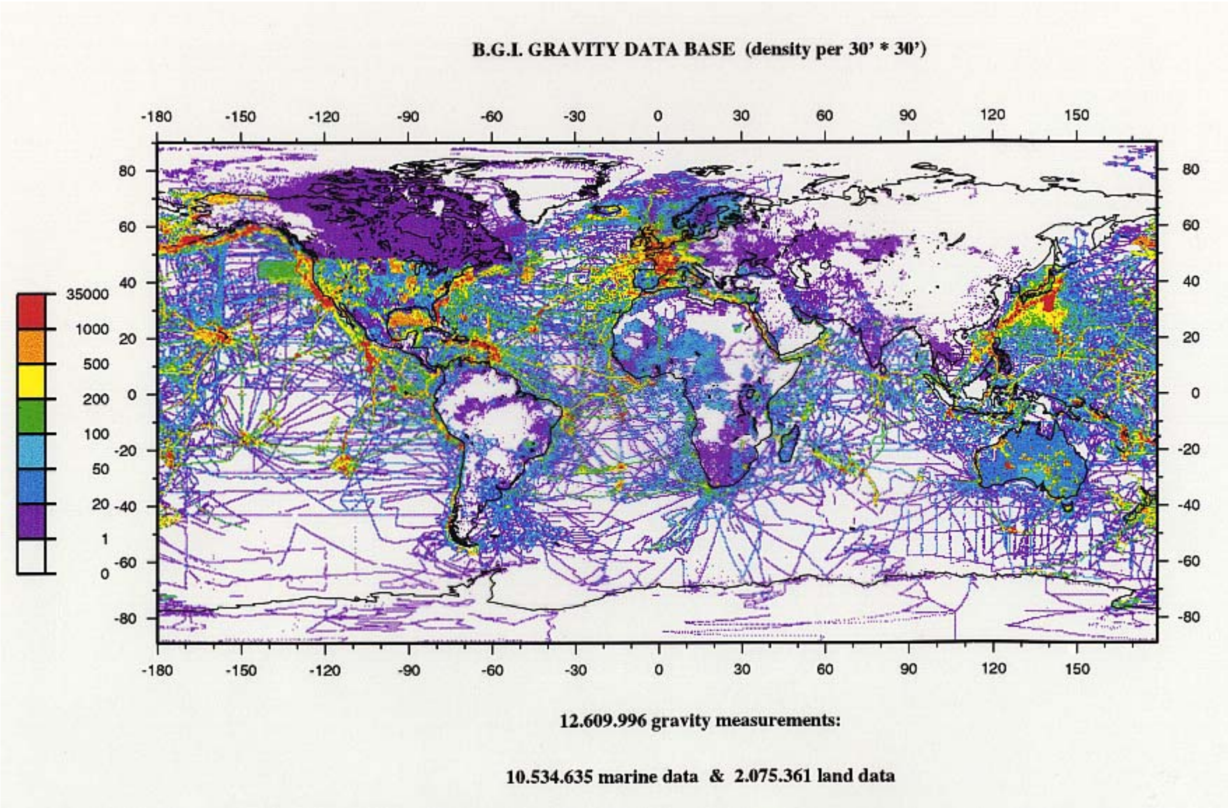


Figure 2.5. *Distribution of presently available measurements of gravity in the data base of the Bureau Gravimétrique International. Over some areas, gridded values only are available. Note that Greenland was recently mapped by airborne gravimetry.*

Gravity vector direction

The direction of the gravity vector is obtained from astronomical latitude and longitude measurements combined with position measurements. The direction is obtained because the astronomical instrument is used with its axis parallel to the plumbline. The differences between the astronomical and the geodetic plumbline directions are denoted deflections of the vertical. They are useful in mountains because their use does not require a precise height. As independent data they are useful for validating other types of measurements or derived quantities.

Gravity gradients

At the earth's surface linear combinations of gravity gradients are observed by torsion balance measurements. These measurements are very sensitive to local mass variations and have therefore mainly been used in prospecting. A few measurements are available today in specific areas.

2.3.2. Derived measurements

Mean gravity anomalies

Point gravity anomalies are used to form mean values of blocks of different sizes such as 5' x 5' or 30' x 30'. Since the gravity anomalies are measured at different heights and in an uneven pattern, their calculation and definition may vary from country to country. In order to carry out the calculation of the mean value (by numerical integration) data gaps are sometimes filled with values computed based on topographic and density information. Such anomalies are called geophysically predicted values.

The most recent set of 30' x 30' mean values has been derived by NIMA (Kenyon and Pavlis, 1996) in the context of the computation of the EGM96 model. Details can be found at :

http://cddis.gsfc.nasa.gov/926/egm96/egm96.html

Spherical harmonic coefficients from satellite orbits

The analysis of satellite orbit perturbations gives us estimates of the coefficients of the spherical harmonic expansion of the gravity potential. The coefficients of low degree and order can be reliably estimated from satellite perturbations only. Global models with coefficients up to degree and order 70 are to-day determined by combining with mean values of surface gravity anomalies (see, for instance, Schwintzer et al., ibid). Much higher resolution models (maximum degree and order 360) also exist, but they are meaningful at such resolution only over areas with adequate surface data coverage. Such models are : OSU91A (Rapp et al., 1991), EGM96 (IGeS, 1997).

2.4. Weakness in gravity field knowledge. What surface gravimetry, airborne gravimetry and satellite altimetry cannot provide

Surface gravity

From section 2.3.1. it may sound as if the gravity field is mapped very well, and this is indeed true for some areas. These areas, at land and at sea, are primarily areas where exploration for resources have taken place. In many land areas state agencies have established a gravity network along highways and geophysical exploration companies then have established denser networks. This has given networks of an uneven character. Measurements are not found on lakes, very scarse in areas with rough topography and almost non existant in polar regions. Areas with shallow water are generally also without measurements because they do not allow ships of sufficient stability to sail a straight course. Data from exploration companies also sometimes have large systematic errors, because these companies have been more interested in the gravity variations than in the gravity values themselves.

Airborne gravity

New airborne gravity surveys are and will naturally be planned so that gravity is measured in a regular pattern such as over Greenland and Switzerland (Klingelé et al., 1996) where such surveys were recently conducted. However the airborne data still suffer errors at the level of 5 mgal for a 30' block. This is not sufficient for many purposes.

Gravity obtained from satellite radar altimetry

As mentioned above satellite radar altimetry may be used to compute gravity anomalies taking advantage of the mathematical relationship between geoid heights and gravity anomalies (Sandwell et al., 1996 ; Knudsen, 1996). Data over sea-ice have also been tentatively used to derive the gravity field in polar areas (for instance by Sarrailh et al., 1997). However, heights obtained from radar altimetry are heights of the mean sea surface and not of the geoid, even if the two surfaces may be close. Detailed comparisons of modern sea-gravity anomalies in areas with moderate ocean currents give typical differences of 1 - 2 mgal. Along the major currents these errors are much larger, up to 10 mgal.

Potential differences

Potential differences are available at all bench-marks. Unfortunately this information is very seldom useful, because the ellipsoidal height is not known. Some countries, like France, have realised the potential use of the information and have carried out ellipsoidal height measurements by GPS at a large number of benchmarks.

Mean gravity anomalies

The calculation of mean gravity anomalies is as mentioned done by numerical integration. The data should be as evenly distributed as possible, and procedures must be used to obtain values at an altitude fixed for each block. This gives problems especially in mountain areas where the data primarily are available along roads, and where the procedures for obtaining an equal height reference require hypotheses about rock densities which generally are not available. So mean values in mountains may have very large errors.

Spherical harmonic coefficients

A large effort has gone into the determination of coefficients of spherical harmonic series. From comparisons of different models, the series exhibit a good stability and have small error estimates for degree up to 25, but then the quality decreases. The reason is very much the uneven distribution of the inclinations of the ad'hoc satellites and their altitude. Most satellites used were not designed for modelling the gravitational potential. Also the analysis of orbital perturbations, which is the basis of this area of research, suffers from the coverage gaps which are inherent to all modes of tracking from the ground. Besides, the minimum altitude of such spacecraft, because of drag, amplifies the filtering effect of the shorter wavelengths of the field and the quality of the surface gravity used to enhance the calculation of the coefficients is varying.

2.5. Range of Goals

2.5.1. Geodesy

One of the main tasks of geodesy is to provide height information used for a variety of scientific and engineering purposes. The gravity potential provides the foundation for the orthometric height systems, which for example have the property that if one point *P* has a height smaller than the height at a point *Q* - and they are both on land in the same datum - then water will flow from *P* to *Q*. An improved gravity field will make it possible to unify the different height systems. The systems differ at present with values of the order of decimeters between two different islands and between an island and a continent having distances up to a few hundred km. Between continents the differences may be up to a metre. The values of these differences should be determined to within 0.05 m from a gravity mission.

Modern (space) methods for positioning and mapping like GPS and SAR (Synthetic Aperture Radar) provide *geometric* heights above the ellipsoid. For most purposes these heights need to be converted to orthometric heights using the simple relationship that the height of the geoid is equal to the difference between the two systems. The geoid height should be known with a precision which do not degrade the precision of the ellipsoidal heights. The geoid height provided by a gravity mission should for this purpose be better than 0.05 m for distances of 100 km.

2.5.2. Satellite orbits

Accurate satellite trajectories are the basis in space geodesy for a variety of applications: precise positioning, monitoring the Earth’s kinematics, mapping the ocean surface by satellite altimetry. The required accuracy has increased over the years, matching the instrumental precision of the tracking devices. All forces acting on a satellite are to be modelled in the process by which an orbit is determined, all of them require better and better models. The gravity field is the dominant perturbation and any mismodelling error shows up in the trajectory, inasmuch as the spacecraft altitude is low.

Positioning and tectonic movements monitoring, and the determination of the Earth rotation parameters can circumvent some of these difficulties by using high altitude satellites (GPS, Lageos 1 and 2), but satellite altimetry uses relatively low altitude platforms which are extremely difficult to locate to one centimetre accuracy in the radial direction as required for ocean circulation studies. The improvement of global models of the gravity field is a necessary step towards the solution of this problem.

2.5.3. Geodynamics

Several basic geodynamical issues related to the dynamics and structure of the Earth's mantle and lithosphere cannot be properly interpreted or even understood because of the inadequate information on the gravity field, considering the accuracy, the resolution and the non-uniform coverage of the available data over the surface of the planet, in particular over continental areas.

The open questions that to be solved require a high resolution gravity mission are related to the structure and composition of the continental lithosphere, where mineral resources of strategic importance are located, and to the dynamics of the oceanic lithosphere at ocean ridges and subduction zones, where the interaction with the other plates is responsible for dynamic processes that control the evolution of important tectonic structures. New achievements in the comprehension of the tectonics of plate interiors and plate boundaries will allow to understand the impact of tectonic forces on the stress field in active seismic regions and to improve our knowledge on the seismic risks in areas where earthquakes are responsible for human and economical losses.

A high resolution gravity mission should yield a substantial improvement of our knowledge of the rheology of the mantle that controls the dynamics of the interior and of the whole Earth; mantle rheology impacts not only the long-time scale convective pattern of our planet but also the interpretation of post-glacial rebound data and present-day sea-level changes. Improved rheological models of the Earth will make it possible to obtain a precise estimate of sea-level changes induced by the response of the planet to the melting of the Pleistocenic ice-sheets. It will thus be possible to retrieve in sea-level signals the contribution originating from climate variations with greater precision with respect to present estimates and to make predictions, on the basis of global climatological models, of the sea-level rise along coastal areas where the population is subject to this natural hazard.

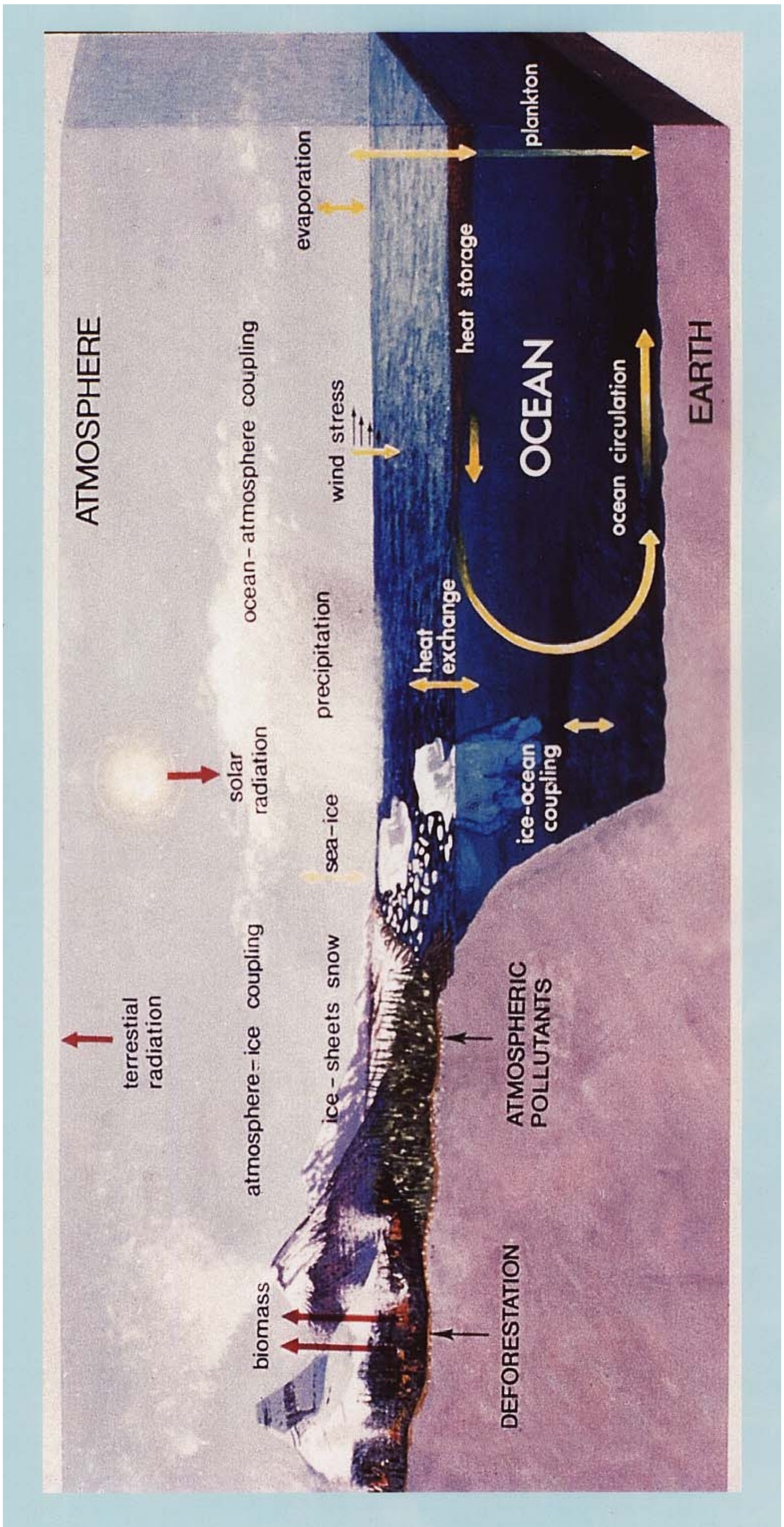
The Earth's gravity field is subject to both short time scale variations from a few seconds to 10^3 yr, and long time scale ones, ranging from 10^5 to 10^8 yr, depending on the geodynamical process responsible for the perturbation of the geopotential and on the response of the Earth's material to such process, that has to deal with the field of mantle rheology. We can divide these processes into two wide categories, characterised by their time scales. The deformation of the mantle and lithosphere following the disintegration of the Pleistocene ice-sheets, the present-day instability of the Alpine glaciers and the co-seismic and post-seismic deformation of the Earth's surface following major earthquakes, fall into the first category of short time scale.

Reorganisation of subduction zones, modification of the density structure of the mantle associated with variations in the convective style modify the gravity field on a much longer time scale and belong to the second category. A high resolution and precise gravity model is of fundamental importance for making a definite advance in our understanding in both categories.

2.5.4. Oceanography

The main application of an improved global gravitational model for oceanography is a better understanding of the mean (or 'absolute') ocean circulation via an accurate determination of the geoid. At the present time, a computation of the 'ocean dynamic topography', is of limited precision because of the multi-decimetric errors in the geoid. If the dynamic topography can be determined at the centimeter level, then the mean strengths of the main ocean currents can be inferred with major consequent benefits to several areas of oceanographic research.

As regards climate-oriented research, a better knowledge of the ocean circulation, and in particular the deep ocean circulation, will lead to significantly improved estimates of transports of the huge amounts of heat, fresh water and salt, and of dissolved quantities including pollutants, and to improved knowledge of the carbon cycle essential to climate studies. Moreover, through numerical assimilation schemes, better knowledge of the mean transports will aid considerably the computations of perturbations in the climate system. The Second (1995) Scientific Assessment of the Intergovernmental Panel on Climate Change has underlined our lack of knowledge of the ocean circulation, its interactions with other parts of the climate system (Fig. 2.6.), and its response to either natural or anthropogenic forcing. For example, the prediction of potential future sea level rise depends critically upon modelling the ocean thermal expansion.



ATMOSPHERE
terrestial radiation
solar radiation
biomass
atmosphere - ice coupling
ocean - atmosphere coupling
ice - sheets snow
sea-ice
precipitation
wind stress
evaporation
heat exchange
ice-ocean coupling
heat storage
OCEAN
ocean circulation
plankton
DEFORESTATION
ATMOSPHERIC POLLUTANTS
EARTH

Figure 2.6. *Schematic of the ocean circulation and its interaction with the atmosphere, cryosphere and biosphere.*

Within 'operational oceanography', a better knowledge of deep ocean currents should in turn lead to improvements in coastal and other shorter spatial scale components of the circulation, through regional assimilation schemes and by provision of improved model boundary constraints. Example developments should include inputs to deep ocean and coastal fishing studies, as currents are key factors for determining biological activity; inputs to improved water quality models for determing the fate of pollutants from rivers and deep-sea dumps; to oil spill models; and to sediment transport and coastal erosion modelling. These examples will be the more relevant where the deep ocean currents impinge near to shelf or coastal regions.

The determination of very precise geoid slopes is crucial to these objectives. Altimetry is now the means by which long term, global monitoring of ocean currents can be achieved. The lack of a sufficiently precise geoid model to accompany the precise altimetry constitutes a major gap in knowledge.

2.6. Proposed Concepts

Two adequate space techniques have been recognised for decades: the satellite to satellite tracking (SST) and satellite gravity gradiometry (SGG). As will be shown below, SST and SGG are complementary: SST is best at providing the long and medium wavelength of the geopotential, while SGG performs best at the shorter wavelengths as a result of the measurement bandwidth characteristics of the accelerometers.

In both approaches, the lower the altitude the better but the more critical is the modelling of the air drag. That is why the satellite must be equipped with a micro-accelerometer which senses the surface forces in the adequate range and frequency bandwidth. A gradiometer, which comprises several accelerometers, can deliver this information from their common mode measurement.

2.6.1. Satellite to Satellite Tracking

This first technique may be viewed as directly coming from the orbit perturbation analysis approach, using at least one low altitude Earth satellite in order to increase its sensitivity to gravitational disturbances which, apart from resonance phenomena, tend to decrease like $\left[R/(R+h)\right]^l$ along a quasi-circular trajectory of mean altitude h. Here R is the Earth's radius and l the wave number (degree) of a spherical harmonic representation of the gravitational potential. This low Earth orbiter (LEO in the following) must be tracked as continuously as possible to extract the gravity signal from the orbital perturbations. The lack of coverage from classical ground tracking is the limitation of this technique. The other problem in tracking from the Earth's surface when high accuracy is required, comes from the propagation model errors, in the atmosphere, of any kind of electromagnetic signal on which the tracking measurement is based. All these drawbacks can be circumvented by tracking the LEO from:

(I) several higher altitude satellites (Fig. 2.7.), such as the ones of the GPS or GLONASS constellation; or

(II) one co-orbiting satellite flying at a mean or variable distance of it (Fig. 2.8.).

The first concept is known as multi-high-low satellite to satellite tracking (one single high altitude spacecraft cannot provide full observational coverage), the second one is known as low-low satellite to satellite tracking.

Figure 2.7. *High-low SST using GPS satellites which orbits are monitored by a network of ground stations.*

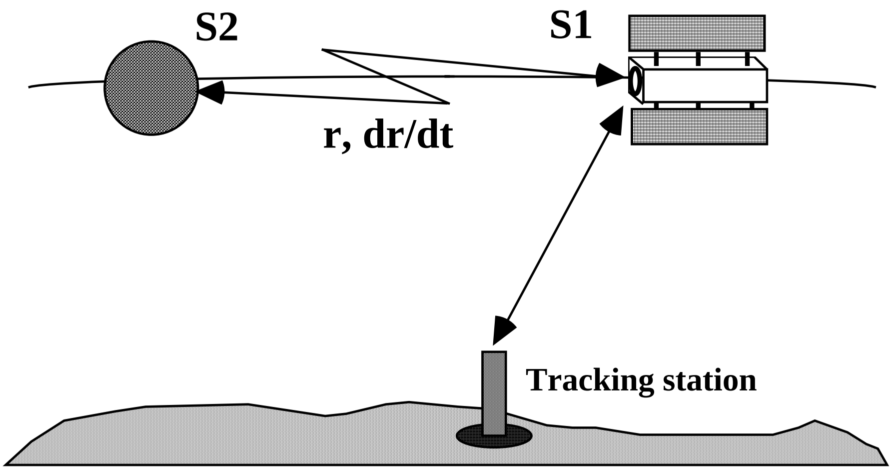


Figure 2.8. *Schematic of low-low SST. One measures the distance r between the two co-orbiting spacecraft, or the range-rate dr/dt. One spacecraft can be also tracked from the ground.*

The global gravity field recovery by, for instance, GPS is based primarily on the combined carrier phase measurement from LEO, and a network of ground receivers which allow the recovery of the GPS trajectories. The International GPS Geodynamics Service (IGS) operates high performance dual-band receivers and is responsible for collecting, archiving and distributing data from this network. Data are processed at analysis centres and precise GPS orbits are available within a few days. The GPS measurements on LEO are used both to determine its orbit and to improve the parameters of the global gravity field in a rather classical iterative process.

In the co-orbiting satellites approach, the basic observed quantity can be a distance or a Doppler frequency shift, or both. This scheme is good at enhancing the sensitivity to short wavelengths of the geopotential, depending on the two spacecraft mean distance. The measurement can be viewed as a direct measure of the relative gravitational potential variations along the orbit at the satellite points but in practice is to be used in a determination orbit improvement scheme for both satellites (preferably using also additional observations of one satellite, for instance from the ground, for proper decorrelation) and for the subsequent derivation of the gravity model parameters.

2.6.2. Satellite Gravity Gradiometry

The second technique measures some derivatives of the gravity vector, called gravity gradients, at a spacecraft in different directions using a gradiometer.

In the free-fall conditions inside a spacecraft, one can measure the difference in the acceleration of gravity between a point where an accelerometer is located, and the centre of mass of the satellite. To make sense of this measurement, one needs to know the precise location of both the accelerometer and the centre of mass. The latter is difficult to do accurately. Also certain forces, acting on the surface of the satellite (drag, solar radiation pressure) will be measured and corrupt the data. Thus one uses two accelerometers rigidly connected and precisely located with respect to each other. Let us assume that their separation is a distance *d*, their sensitive axes (the direction in which they measure acceleration) are aligned parallel to the *x* direction, and the line between them has the same orientation as the sensitive axes (Fig. 2.9.). Then with *V* being the potential function :

$$V_{xx} = (a_2 - a_1)/d \qquad \text{(to first order in } 1/d\text{)}$$

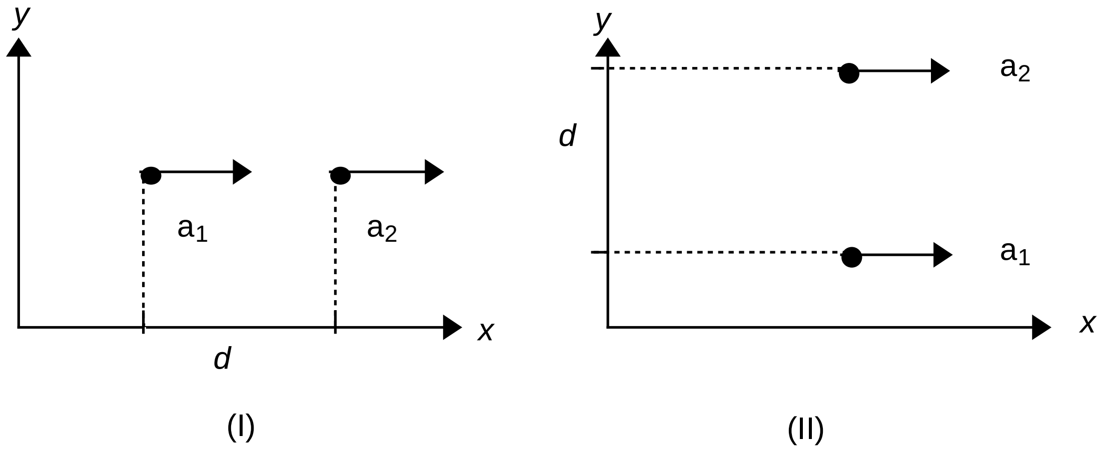


Figure 2.9. *Principle of gradiometry based on pairs of accelerometers. In (I), one measures:* $(a_2 - a_1)/d \approx V_{xx}$ *and in (II):* $(a_2 - a_1)/d \approx V_{yy}$ *.*

Similarly, if the separation is along the (orthogonal) direction *y* , the same calculation gives the cross gradient V_{xy} (which is the same in value as V_{yx}), etc. The sum of the three “in line“ components (V_{xx}, V_{yy}, V_{zz}) is zero, which is called Laplace’s equation. Therefore one has five independent components. A complete gradiometer will measure all gravity gradients. In reality, the satellite, and the instrument in it, rotates. The result is that what is actually measured is gravity, not gravitation (this situation is similar, but more difficult, to that encountered in physical geodesy, when dealing with “Earth-fixed“ data). Gravity gradients are highly sensitive to the local features of the field in the proximity of the measurement location. This is why in the past terrestrial gradiometry has been applied in exploration geophysics, using torsion balances. For the same reason a spaceborne gradiometer of a given accuracy will result in a more successful mission if it is in as low an orbital altitude as possible. Tests with airborne gradiometry were carried out some years ago with modest success. In space the high sensitivity of a gradiometer for spatial details of the Earth's gravity field should make it ideally suitable for gravity field refinement. Naturally one needs to know where the measurements are taken (the orbit). No actual experiment has been carried out so far.

2.7. Missions Proposed in the Past

The requirements and limitations of global Earth's gravitational potential models have been recognised regularly over the last decades. SST was first used in mapping the near side gravity field of the Moon (Earth being considered a satellite of the Moon) and quickly picked up by the Williamstown Conference (1969) as the technique of choice for mapping Earth's gravity field. It was demonstrated successfully between the Apollo Spacecraft and one Applied Technology Satellite, ATS, then between the 24 hour satellite ATS6 and the GEOS 3 altimetric satellite but without much scientific result due to the relatively high altitude of the lower satellite. Groups engaged in SST methods then favoured the low-low mode, in which two orbiters flying very close circular trajectories would have performed relative velocity measurements of high accuracy, at the lowest possible altitude. The United States GRAVSAT project originated from it. Technical difficulties and budget constraints kept postponing the start of the project and finally led to its termination. The next project, named GRM, was to map the Earth's gravity and magnetic fields using two drag-free spacecraft orbiting in polar orbit at a very low altitude. The fate of GRM was similar to GRAVSAT and the project disappeared in 1987.

At that time, based on the success of the CACTUS micro-accelerometer experiment on board the D5B satellite during 45 months, the French GRADIO project was under study. It was an SGG experiment aimed at measuring the full set of gravity gradients (and separating it from the spacecraft attitude disturbances) by means of eight three axis micro-accelerometers of a new generation. The project studies served as a basis to the Agency ARISTOTELES project (ESA, 1989). For the first time, high-low SST using GPS, and SGG were to be combined. ARISTOTELES, which could have been realised as a joint ESA-NASA venture, was not pursued.

Then, in the framework of the co-operation between NASA and the Centre National d'Etudes Spatiales (CNES - French Space Agency), the Gravity and Magnetics Earth Surveyor project, or GAMES was envisaged, in which two co-orbiting satellites at about 200 km altitude would have performed precise laser measurements of their mutual distance; one spacecraft would have been equipped with a GPS receiver to better determine the first part of the gravity spectrum; the magnetic field would have also been measured.

The French BRIDGE project was not so ambitious and its goal was to provide an improved knowledge of the geopotential in its long and medium wavelength part. It would have consisted of a low free-flyer carrying a DORIS station so as to link with satellites such as SPOT 5 or ENVISAT which will carry second generation DORIS receivers. Simulations have shown that such a concept could improve the present global gravity field models.

None of these projects became a reality but all the gravity objectives and goals which were elaborated for them are still valid. In trying to accommodate instruments on a single platform to map the two geopotential fields, great difficulties have been encountered entailing a great degree of complexity and increased cost. The magnetic field has first been measured from space during the NASA MAGSAT mission in 1979; for the refinement and for the determination of secular variations the Danish OERSTED and the German "**C**atastrophes and **HA**zard **M**onitoring and **P**rediction" (CHAMP) missions offer new opportunities.

2.8. Current proposed missions

Other concepts have now emerged, some very ambitious and targeted at this domain of geophysics, some others dedicated to other areas of science but which could serve our purpose, and others not so ambitious but aimed at the improvement of the low and medium frequency parts of the geopotential spectrum. All techniques considered in these projects are assumed to deliver measurements or yield the knowledge of position or velocity of one spacecraft, or inter-satellite distance or velocity in the case of multi-spacecraft mission scenarios, or gravity gradient components, or any combination of these observables, with given error spectra. Table 2.1. summarises the characteristics of the projects which have been recently under investigation in various agencies.

Table 2.1. *List of Proposed Missions*

Name	Agencies	Techniques	Mean Altitude (Km)
STEP	ESA-NASA-CNES	SST/GPS, SGG	400-350
CHAMP	DARA, GFZ, CNES, JPL	SST/GPS (+ Laser)	450-250
GOCE	ESA	SST/GPS, SGG	260-270
GRACE	NASA (JPL), GFZ	Low-Low SST (+ GPS)	450-250

STEP : Satellite Test of Equivalence Principle
CHAMP : Catastrophes and Hazards Monitoring and Prediction
GOCE : Geophysics and steady state Ocean Circulation Explorer: this project.
GRACE : Gravity Recovery and Climate Experiment

The STEP project would realise several experiments in fundamental physics, especially one aimed at verifying the principle of equivalence between the inertial mass and the gravitational mass based on differential micro-accelerometer measurements using proof-masses of different materials. A medium altitude of about 400 km is proposed, which is fairly high with respect to our needs but the extreme accuracy of the gravity gradient measurements could compensate for this.

Although the CHAMP mission is not focused on gravity, its goals are similar to those of BRIDGE as regards the geopotential. CHAMP is a German approved mission, to which France and the USA contribute, which will be launched in mid-1999. CHAMP will essentially perform a magnetic field and a limb sounding experiments. GPS receivers and a micro-accelerometer will make it a good candidate for gravity field mapping. GRACE or GOCE, if realized, will provide significantly higher resolution and accuracy than CHAMP.

3. RESEARCH OBJECTIVES

3.1. Geodesy (Accurate heights, unification of height systems, local/regional).

The results of a gravity field mission may be made available to the users in different forms. The most important are :

a) filtered measurements of components of the gravity vector or the gravity gradient matrix given at satellite altitude,
b) derived surface mean gravity anomalies or mean geoid heights
c) coefficients of a spherical harmonic expansion of the gravity potential, V.

These results may be based on satellite measurements (SST,SGG) only, or obtained by combining other satellite measurements or ground data. The result may be used to achieve the following geodetic goals :

3.1.1. Uniform world-wide height datum

The determination of height or elevation requires the knowledge of an equipotential reference surface. Such reference surfaces are defined differently from island to island, from region to region and from continent to continent. The definition is made by assigning a value to a reference marker (a bench mark). The value chosen is normally based on observation of mean sea level for a given epoch at a number of tide gauges. Sometimes the definition involves even a model of the time-dependence of the land movements for the epoch used to define the mean in order to eliminate inconsistencies between different tide-gauge mean values and the levelling lines which connect the tide-gauges. Originally this procedure was supposed to assure that different areas used the same reference surface, but it was realised that sea-level changes, the existence of a stationary part of the sea surface topography and changes in coastal currents and land movements gave each region its separate height datum. For instance, the height datums at the islands of Eastern Denmark and Northern Germany have been compared by hydrostatic levelling (Andersen et al., 1990) and a height difference of 7.6 cm was found between these two regions. Also, there is a sea level difference of 25 cm between the Atlantic and Pacific coasts of Panama. The datum differences cause practical and scientific problems :

a) it is not possible to separate land movements and sea-level changes (by having a link to a number of points of "no"-movement),
b) heights needed for some engineering works (e.g. bridges, tunnels connecting islands or different continents) need to be unified before construction,
c) the height references used for hydrographic mapping and tidal modelling are inconsistent,

d) the mean sea surface at tide gauges is recorded in different systems, which makes it impossible to use the values in global studies of mean sea level.

And then there is naturally the main problem, namely that the height datums are only physically realised on land. We also need a height reference surface at sea primarily for oceanographic reasons, see section 3.4.5.. However, if a gravitational model is available, one may use this at any point with known position to calculate the gravity potential W. Then the geoid is found to a good approximation along the vertical at a distance $(W\text{-}W_o)/g$, where W_o is the value fixed for the potential of the geoid. The calculation may be improved by augmenting the gravity model with known regional gravity information. This procedure may be executed in points with known orthometric heights and thereby used to determine ΔH_o (see 2.3.1). Using a gravity model alone, we need to have a global height datum realised with an error below 10 cm.

3.1.2. Transformation between GPS derived ellipsoidal heights and orthometric heights

The Global Positioning System (GPS) makes it possible to determine ellipsoidal height differences $\delta h = h(P) - h(Q)$ between two points P and Q. For nearly all practical purposes these heights must be converted to orthometric heights. If one of the points is a point with a known orthometric height $H(Q)$, and the geoid height difference $\delta N = N(P) - N(Q)$ is known or may be calculated from a model, then $H(P) = H(Q) + \delta N - \delta h$. There is an urgent need to provide improved geoid information as can be seen from recent efforts to determine regional geoids in Europe and North America. These regional geoids are slices of equipotential surfaces, which enable the calculation of geoid height differences, but not of the "absolute" geoid height. These geoid height differences have a precision of a few centimeters over 100 km in well surveyed regions, but the error increases to several decimeters in Alpine-like areas. A large effort in the USA (Nerem et al. 1995) has been completed recently, which globally aimed at improving the geoid by collecting all available gravity information with the purpose of constructing a spherical harmonic expansion to degree 360. This effort has improved the current estimates of the spherical harmonic coefficients (IGeS, 1997), but it can obviously not make up for the missing gravity information or for the systematic errors contained in gravity data computed on the oceans from satellite radar altimetry, see section (2.4.).

The heights are needed for numerous practical as well as scientific purposes. Height systems may be established without waiting a decade for obtaining a value of mean sea level, which has changed meanwhile. Orthometric height differences may be calculated from ellipsoidal heights in land areas not connected to coastal tide-gauges for example to be used in irrigation projects.
In mapping projects using airborne or satellite sensors, the height information is given in terms of ellipsoidal heights. Here the geoid is needed to convert the heights to orthometric heights. It is a very useful product for developing countries, making the mapping process fast and more economical. For global mapping projects (e.g. from space) a geoid better than 1 m is needed. For technical mapping using airborne SAR (Synthetic Aperture Radar) a geoid with decimeter accuracy is needed in order to fully exploit the SAR capabilities.

For scientific purposes ellipsoidal height differences need to be converted to orthometric height differences in order to study vertical land movements (the older measurements being given as orthometric heights). Levelling done on ice-caps may also be compared to new values obtained from GPS, if we can convert the ellipsoidal heights to orthometric heights. Precisions needed here are at the cm level. If an accurate high resolution global spherical harmonic model is combined with local gravity and topographic data, precise height differences may be calculated everywhere.

3.1.3. Ice and land monitoring combined with geoid knowledge

As mentioned in 3.1.2. the monitoring of vertical ice and land movements have until now been done by comparing repeated orthometric heights or potential differences obtained from precise levelling. These comparisons are typically done with 50 year intervals. GPS now give the possibility for cheaper and faster measurements. However in order to preserve these invaluable time series, the ellipsoidal heights must be converted to orthometric heights. Here again a precise global gravity field model is needed as a basis for regional or local geoid models. The global model will contribute with information which makes the local geoid solutions unbiased. The importance of understanding the slopes of the ice sheet surfaces to models of ice sheet dynamics, and thereby to global sea level change studies, cannot be underestimated. The IPCC report (1995) makes clear that the uncertainties in global change sea level estimates for the past and next centuries owing to lack of knowledge of the ice sheets are very large.

3.2. Altimetric Orbit Considerations

First, there are still errors of the order of 1 to 2 cm in the orbit of the most precise altimetric mission TOPEX/POSEIDON (T/P), due to the uncertainty of present potential coefficient models. Part of this error is ”geographically-correlated”, meaning that it occurs at constant geographical positions , so that it is lumped into the altimetric height measurement and the estimates of the ocean mean dynamic topography. Such centimetric errors are equivalent to large values of ocean transports (see section 3.4).
Second, 2 cm orbit accuracies cannot be obtained for satellites at altitudes lower than that of T/P without a significant improvement in the gravity model. For example, Tapley et al. (1995) demonstrated that the Geosat Follow On (GFO), which is to fly at an altitude of about 800 km in 1997, would have a radial orbit error contribution of approximately 3 cm from gravity field mis-modelling (the largest component of its orbit error budget), whatever the satellite tracking technique. Future, cheaper, satellite altimeter missions at lower altitudes are thus not feasible if T/P quality orbits are required.

Third, data derived from previous missions which lacked the most modern tracking systems will become all the more valuable as the gravity model is improved and their orbits are recalculated with that model.

3.3. Geodynamics and time variations of the gravity field

A high resolution gravity mission will allow the solid Earth geophysical community to make a definitive progress in the physical understanding of both short time-scale geodynamical processes, which can interfere with the activities and life of the human beings, and long time-scales ones, which are relevant to the comprehension of the dynamics of our planet; how this can be the case, is discussed in the following.

3.3.1. Short time-scale geodynamic processes. Post-glacial rebound and sea-level changes

An important geodynamical process of our concern on time scales of 10^3 yr is post-glacial rebound the signatures of which are still visible in the continental lithosphere that supported the load of the ice-sheet, before deglaciation terminated about 7000 years ago. Melting of large ice-sheets in the northern and southern hemispheres during the Pleistocene has modified the state of isostatic equilibrium that the Earth is still trying to restore by filling with mantle material, flowing from equatorial to polar regions of the mantle, the deficit of mass underneath the deglaciated areas (Spada et al., 1990). This ongoing flow is of course affecting the gravity field on time scales comparable with the life-time of the human beings; a high resolution gravity mission can help quantify these short time-scale variations of the gravity field: one cannot monitor the gravity changes due to mantle processes during the mission itself, but one can make use of the new gravity data to improve the rheological models that allow the prediction of the present-day changes of the gravity field. The way in which an advanced gravity mission will provide important constraints on the short time-scale component of the varying geoid and gravity due to fast mantle processes, is by allowing a detailed *knowledge of the rheology of the mantle* and by realising a *reference* to which future measurements will be compared.

The post-glacial rebound processes induce a gravity signal which is clearly discernible as large negative free air gravity anomalies in previously glaciated areas. First, modelling of post-glacial rebound allows to constrain the viscosity profile of the mantle from the comparison of the gravity field monitored from a satellite mission with model results. Today time variations of the gravity field and geoid depend on the other hand on mantle viscosity; learning about the viscosity structure allows to make predictions on the temporal variations of the gravity field. Second, since present-day sea-level changes are also affected by post-glacial rebound, present-day deformation of the solid Earth affects sea-level variations, which must be distinguished from the signal due to the temperature variation of the oceans and contributions from the melting of alpine glaciers, related to climate changes. Post-glacial rebound induces a sea-level signal because the deformation of the solid Earth following the melting of the huge Pleistocenic ice-sheets affects in a different fashion the geoid and the surface of the solid Earth; variations in the distance between the mean sea level and the surface of the solid Earth induced by the readjustment of the planet to the melting, is visible as a sea-level signal.

Studies based on radially stratified Earth's models have already clearly indicated the profound implication that mantle rheology has on the prediction of sea-level changes due to the response of the solid Earth to the melting of the Pleistocenic ice-sheets (Mitrovica and Davis, 1995). Fig. 3.1. provides a schematic picture of how the shape of the sea-surface during the built up and melting of the ice-sheets, pictured by the grey regions, is affected by the rheological properties of the mantle, comparing the behaviour of a rigid planet (a) with that of a viscous mantle (b). With respect to the equipotential V_o that defines the sea-surface before the built-up of the ice sheet, represented by the short-dashed curve at a constant distance from the bottom of the sea, the gravitational attraction of the mass of the ice-sheet is responsible for the pull up of the equipotential, denoted by the continuous curve V_F. Sea-level rises in proximity of the ice-sheet and falls away from it; the long dashed curve denotes the sea-level, corresponding to the equipotential V_* that one would expect because of the extraction of the water from the oceans to build the ice-sheet, responsible for a sea level fall from V_o *to* V_* .

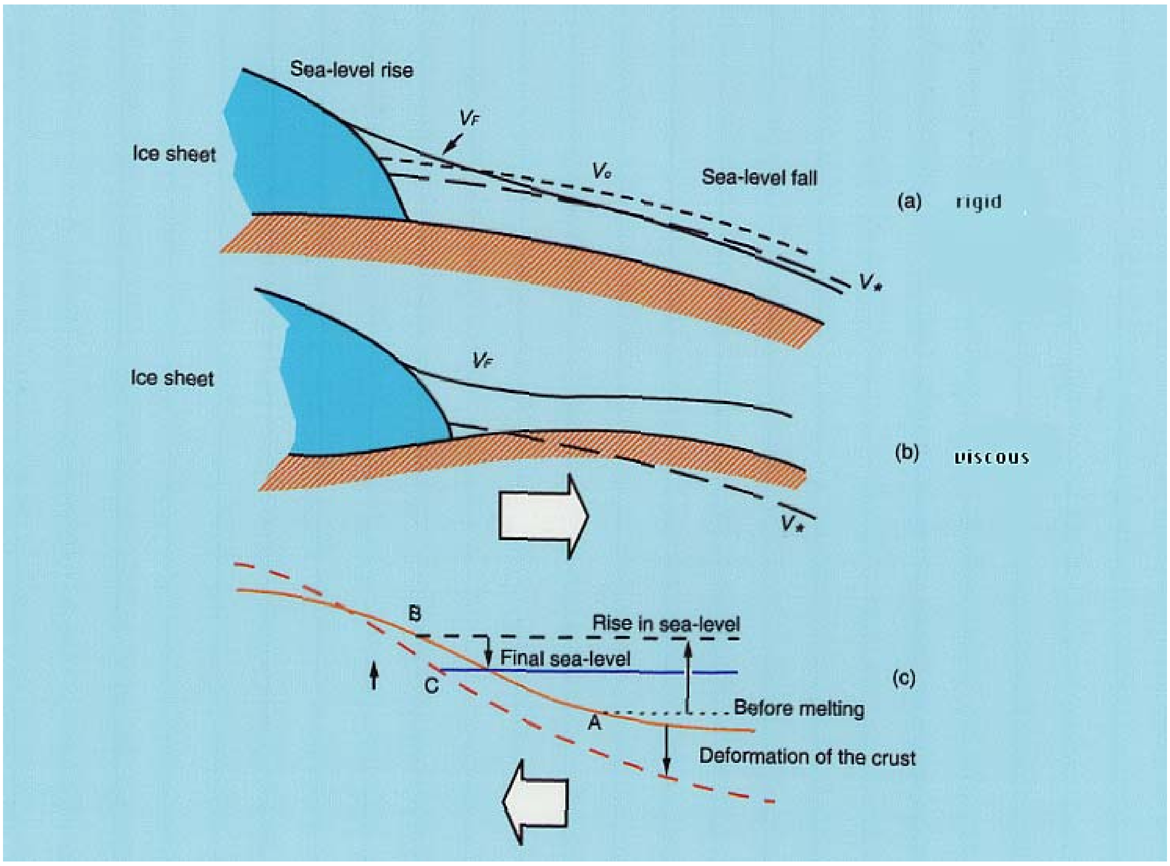


Figure 3.1. *Sea-level variations following the built up of an ice sheet for a rigid mantle (a) and a deformable viscous one in (b). The short-dashed curve denotes the equipotential sea-surface before glaciation. The long dashed one corresponds to the sea-surface after the extraction of the water without the gravitational effects of the ice mass. The solid curve accounts also for the gravitational effects of the ice-sheett. Panel (c) shows the expected sea-level variations along a continental margin away from the ice-sheet following deglaciation.*

The effects of the rheological behaviour of the mantle are portrayed by the deformation of the shaded region that is displaced downward due to the ice load. Mantle material flows from a zone beneath the load towards the periphery, as shown by the white arrow; this process, coupled with the reduction of the water load in the ocean, induces the upwelling of the oceanic crust at some distance from the ice load. In comparison with the rigid mantle of panel (a), the shape of the equipotental sea-surface is now controlled not only by the gravity field of the ice-sheet and water, but also by that of the deformable viscous mantle. The deviations of the continuous curve with respect to the dashed one now differ substantially from those of panel (a); in particular, the upwelling of the oceanic crust at distance from the ice-load is responsible for a quite different sea-level signal at distance from the ice-sheet in comparison with the case of a rigid mantle of panel (a). This simplified picture clearly indicates the major role of mantle rheology in controlling the shape and the time-variations of the sea-surface during the occurrence of ice-sheet built up; similar arguments are valid of course also for the phase of ice-sheet melting.

Another example that illustrates the importance of the deformation properties of the mantle in controlling the sea-level is shown in panel (c), which illustrates the sea-level changes following the deglaciation along a continental margin far from the ice-sheet. Before the water is added to the oceans because of ice-sheet melting, the shoreline is located at point *A*. The water added to the oceans is responsible for a fast sea-level rise along the coast from *A* to *B*. The water load causes the subsidence of the oceanic crust, and an uplift of the continent because mantle material is flowing from beneath the ocean to beneath the continent. This causes a sea-level drop along the continental margin from *B* to *C*, accompanied by the tilting of the margin. The time dependence, and the amplitude of the response of the mantle beneath the oceanic and continental crust are ultimately controlled by the rheology of the mantle, which clearly indicates the importance of studying the deformation properties of the mantle for sea-level predictions.

In short, learning about the gravity signal due to geodynamical processes varying on short time scales from 10 to 10^3 yr, allows to estimate the rheological parameters of the asthenosphere and mantle and to predict how the gravity and geoid are changing at present and in the near future due to post-glacial rebound. A precise estimate of the sea-level signal due to the rebound process will allow to quantify the contribution which originates from climatic variations.

A high resolution gravity mission can help us improve the radially stratified models of the Earth that have been used until now in post-glacial rebound studies on the gravity field. An improved rheological model which accounts also for the effects of lateral variations in the viscosity will be used to produce refined predictions on vertical motions due to the rebound process and better estimates of sea-level variations. Seismic tomography indicates that the structure of the shields where deglaciation occurred is far from being that of the 120 km uniformly thick lithosphere commonly used in rebound models, supporting the evidence of deep and cold continental roots where the viscosity is much higher than in the standard reference upper mantle (Ricard et al., 1991). Forward modelling of the effects of these roots on the gravity signals due to the melting of a reference ice-sheet indicates that, in order to resolve the radius of the root to a resolution of 200 km and a viscosity contrast of two orders of magnitude between the root and the normal mantle, the required accuracy in the gravity signal is $\delta g = 1$ mgal with a resolution (half wavelength) of about 400 km. Higher resolution in the determination of the lateral extent of the high viscosity region beneath the continental shields would require higher accuracy and resolution of the gravity.

It should be noted that only a mission of the GRACE or GOCE type (see section 2.8) will allow to reach everywhere such a high accuracy at the horizontal resolution of 400 km.

3.3.2. Long time-scale geodynamic processes. Structure and dynamics of the lithosphere, oceanic and continental, and of the upper mantle

Although the gravity pattern recorded during a short lived mission represents a single snapshot of the long-term evolution of the dynamic processes involving the interior of our planet, one can nevertheless gain new basic achievements, also in the dynamics of the processes, from an improvement of the accuracy, resolution and world coverage of the gravity field. In particular one can expect big progress in our understanding of the structure, evolution and dynamics of the oceanic and continental lithospheres and of their interaction with the upper mantle.

The oceanic lithosphere

Among the most important structures that involve the oceanic lithosphere we find the trenches, whose dynamics can be constrained by the specified mission requirements. These structures characterize the region where the old oceanic lithosphere starts to subduct into the mantle, and are certainly the most interesting to study because of the complicated geodynamical processes that occur in the trench regions, due to the bending of the oceanic plate, its interaction with the viscous upper mantle and the dynamic behaviour of the overriding continental plate.

An important issue, that is mostly related to the deformation style of the converging plates at subduction zones, is the time evolution of the basins in the back arc region. In particular, the scientific community is interested in gaining new insight into the time evolution and formation of these basins, into the deformation style during the geologic time, into the amount of crustal and lithospheric thinning and amount of tectonic subsidence due the cooling of the basement with respect to the subsidence driven by the subducting plate (Bassi and Sabadini, 1994). Required performances for resolving the dynamic structure of these regions, controlled by deep density contrasts and convergence velocity of the plates, is a gravity field accuracy of a few milligals and a resolution (half wavelength) of 100 km.

The subducted oceanic lithosphere is denser than the surrounding mantle, and this causes the sinking of the plate. In doing so, the subducted lithosphere induces a downward flow in the mantle which causes a subsidence in the back-arc basin and uplift of the lithospheric wedge in proximity of the trench region. It is thus clear that this sequence of subsiding and uplifting regions is dynamically maintained out of isostatic equilibrium by the mantle flow driven by the subducting plate; a precise knowledge of the gravity field at subduction zones can thus help constrain the dynamic process of subduction. In particular, subduction zones are characterized by two wavelengths, associated with the broad back-arc basin and the narrower uplifting arc and trench, of a few hundreds of kilometers and of about 50-100 km respectively.

One of the major issues which must be understood at subduction zones is the great variability within the subducting plate of the depth of the transition zone between the state of extension that dominates at shallow depths and the state of compression at great depths, as inferred from deep seismicity. Worldwide homogeneous, high resolution and accurate gravity data will help clarify this issue, shedding new light on the dynamic interaction between the subducting plate, the mantle and overriding plate that controls the state of stress, and the gravity anomalies, in the trench region. Required accuracy and resolution are 1-10 mgal and 50-100 km.

Studies on mid-oceanic ridges, seamounts and hot-spots, which are other important features characterizing the structure and dynamics of the oceanic lithosphere, will also benefit from the acquisition of new, high resolution gravity data. A deeper knowledge of the active zones of the ridges, where the oceanic lithosphere originates and spreads laterally, will enhance our understanding of one of the basic processes of plate tectonics, which is the initiation and time evolution and growth of the oceanic lithosphere.

The 420-670 km transition zone

An improved gravity field model will also be useful for studying the transition zone in the Earth mantle at depths between 420 and 670 km, where the boundary between the upper and lower mantle is located.

Recent findings on the structure of dense oxides, derived from seismic tomography and laboratory creep experiments on mantle materials, suggest that these materials between the 420 and 670 km mantle discontinuities could be much stiffer, or more viscous, than usually considered in the standard rheological models of the Earth. Preliminary studies on the effects of viscosity variations between 420 and 670 km have demonstrated that results from a high resolution gravity mission can be used to constrain the viscosity increase at the transition zone. These studies are based on the analysis of the gravimetric signal induced by post-glacial rebound, and use Earth mantle models where the 420 and 670 km discontinuities are properly taken into account. The results, obtained for post-glacial rebound in North America, can be extended to other continental areas such as the Baltic, Siberian and Antarctic regions. Results on the structure of the upper mantle, derived from the modelling of the short time-scale process of post-glacial rebound, can also be used to improve the physical modelling of the long time-scale subduction process, where the tip of the slab interacts at 420 km with the high viscosity region. The stiffening of the upper mantle at the transition zone is responsible for a quite different gravity pattern recorded in continental areas by the gravity mission when compared with the uniform upper mantle, because the hard viscosity layer is able, with respect to the homogeneous viscosity case, to dynamically support the subducting slab.

Studies on the joint inversion of gravity and seismic tomographic data (Zerbini et al., 1992) have shown that the combined use of these two data sets (which reduces the mathematical instabilities characterizing the inversion procedure) improves significantly the images of the interior of the Earth based on the inversion of tomographic images alone. These studies, performed on upper mantle structures such as subduction, have shown that the required accuracy and resolution are 1-2 mgal and 100 km respectively. Coupling these findings with the above remarks on the sensitivity analyses performed for lithospheric and upper mantle structures, it is thus clear that global high resolution gravity data will greatly improve the images of the internal structures of the Earth, from the surface to a depth of 670 km.

The continental lithosphere

Although the continental lithosphere is somehow passive when compared with the dynamically active oceanic one, which is pushed at ocean ridges and pulled at subduction zones, it is nevertheless the locus of important geodynamical processes, due to collisions with other continental and oceanic plates and to interactions with the asthenosphere and upper mantle. This coupling with the asthenosphere and mantle is due to thermal processes, such as hot-spots, diapirs or small scale convection instabilities, responsible for continental rifting and formation of sedimentary basins.

Sedimentary basins in continental areas which occur away from the direct influence of plate margins processes are the signatures of the response of the continental lithosphere to extensional forces and thermal instabilities in the asthenosphere or upper mantle. The outcome of a high resolution gravity mission can play a crucial role in understanding the formation of these basins, because the thermal instabilities, through the associated density anomalies, will be visible in the measured gravity field, provided that the resolution and accuracy are 1-2 mgal, 50-100 km respectively. Besides the scientific importance of the studies of sedimentary basin formation, there is an economical interest in strategic resources located in the basins, such as oil fields.

Another important process that occurs in the continental lithosphere is rifting, associated with elongated depressions where the entire thickness of the lithosphere is deformed by extensional forces. Rifts are common tectonic features because the strength of the continental lithosphere is least under extension, and can be found in a variety of tectonic settings. Rifts are often associated with upwelling of hot material from the mantle, visible in the gravity field because of its negative density with respect to the normal mantle. An improved gravity field over continental rift areas, determined with an accuracy of 1-2 mgal and a resolution of 20-100 km, will enhance our knowledge on the major geodynamical causes of rift formation and evolution.

A full comprehension of the dynamics of these tectonic mechanisms is relevant for the correct interpretation of other important tectonic processes such as tectonic uplift that impacts sea-level trends along continental margins. Tectonic uplift can be due to active convergence between plates and density anomalies embedded in the upper mantle, which can be resolved with an accuracy and resolution of 1-2 mgal and 50-100 km respectively. For example in the Aegean region, the tectonic uplift of the island of Crete of 2-3 mm/yr, is caused by subduction and active convergence of the African and the continental Aegean plates. It is responsible for a sea-level signal, superimposed to the eustatic and isostatic ones due to Pleistocenic deglaciation.

These few examples show how a better knowledge of the gravity field will improve our understanding of the dynamical interactions between the continental lithosphere and upper mantle, that cause the formation of important tectonic structures in the continents. Also the correct interpretation of sea-level changes along continental margins subject to active tectonics may require, at least in certain continental areas, an improved gravity model.

Previous studies based on sensitivity analyses (Zerbini et al., ibid.) have demonstrated how accurate high resolution gravity data can constrain the structure of density anomalies beneath the continental lithosphere, and shed new light on the origin, time evolution of thermal anomalies which are relevant for the dynamics of small scale convection and diapiric processes in the upper mantle. These processes, as stated above, affect the state of stress and the style of deformation of the continental lithosphere, such as in continental rifting zones and sedimentary basins. These studies have been based on the forward modelling of the gravity signal induced by anomalous density structures embedded in a viscoelastic lithosphere and upper mantle, compared with the results of an homogeneous reference model. The parameters characterizing these anomalous sources have been chosen in the models in agreement with those expected for small scale convection and diapiric processes in the upper mantle. It was found that the gravity signal due to anomalous density structures underneath the continental lithosphere is very sensitive to the thickness of the crustal elastic layer, to the depth of the anomaly, to the rheological properties of the lithosphere and to the time history of the source; in principle, these parameters can be derived from the gravity field. High resolution in the source parameters is expected only for depths of 150 km at most; for deeper sources, typically from 300 to 500 km, the resolution deteriorates. The possibility of resolving also the characteristic time constant of upper mantle sources indicates that not only the shape but also the dynamics of the thermal instabilities at the base of the lithosphere or in the upper mantle can be understood; the required accuracy and resolution that are necessary to reach these goals are δg = 1 to 2 mgal and 100 km respectively.

The possibility that the scientific community has to understand basic phenomena, such as the thermal evolution and style of deformation of the continental lithosphere, including the petrological aspects besides the dynamic ones, is thus strongly dependent on the availability of new, well distributed and high quality gravity information over continental areas.

3.4. Ocean Circulation and Sea Level Changes

3.4.1. Introduction

Knowledge of the ocean's central role in modifying climate, through its large heat capacity and transports and the complexity of its interactions with the atmosphere and cryosphere, has long been insufficient for accurate prediction of climate change as a result of fluctuations in natural or anthropogenic forcings. For example, it is known qualitatively that half or more of the excess energy input (the incoming solar radiation minus the infrared radiation to space) in tropical areas is carried by the oceans towards the poles, the other half being tranported by the atmosphere. However, quantitative estimates are coarse, and predictions of how such fluxes would be modified by 'enhanced greenhouse forcings' are even coarser. Such uncertainties resulted in the formation of the World Climate Research Programme (WCRP) by the World Meteorological Organisation and the International Council of Scientific Unions. This question has been and is being addressed through very large oceanographic research programmes, like the World Ocean Circulation Experiment (WOCE) and the Climate Variability programme (CLIVAR).

These programmes rely heavily on the availability of satellite altimetry data such as provided by the TOPEX/POSEIDON and ERS-1/ERS-2 missions which are to be followed by JASON and ENVISAT around year 2000 ; these satellites allow the measurement of very precise, regular and quasi-global sea surface heights. As most changes in ocean surface currents on timescales of a few days or longer result in geostrophic balance, gradients of the sea surface pressure (or the ‘dynamic topography’, i.e. the sea level above the geoid) can be employed almost directly as proxy-current information. Unlike in situ measurements, they are global, synoptic and can be repeated for many years. Unlike other quantities measured from space, they are related to ocean processes and currents within the whole water column. They are also of simple oceanographic significance so that they can be assimilated directly into ocean and climate numerical models.

While *variations* of the sea level and thus of the ocean currents can be derived directly from satellite altimeter data, the *absolute* value of the ocean dynamic topography requires the independent determination of what would be the elevation of an homogeneous ocean at rest, that is the geoid. The latter is not known at present with sufficient precision. Indeed the typical elevation scale of the dynamic topography is of the order of 10 cm to 1 m, while the precision of present geoids is of the order of several decimeters on the scale of ocean circulation features.

The need for a more accurate geoid is also important at the coast, as many countries have performed conventional geodetic levelling related to the local sea level. Because of the large scale permanent ocean circulation, this local sea level is not the same everywhere. Global connection of all the levellings of the world thus requires the determination of an accurate reference surface. The purpose of a dedicated gravity mission is to provide such a geoid reference level for the whole world for application to a wide range of scientific studies in which sea level data are employed. Better knowledge of the absolute ocean circulation will be most obviously applicable to oceanographic and climate research. It will also provide input to systems in which dynamical numerical models are adjusted to observations through data assimilation techniques, and then used for deriving ocean quantities such as heat transports, or for making predictions of ocean currents over different timescales for scientific and operational purposes.

3.4.2. Global Sea Level Measurements

Measurements of sea level are made from space via satellite radar altimetry, and from in-situ devices such as coastal tide gauges, bottom pressure recorders and GPS-buoy systems. During the last decade, the technique of radar altimetry has become a fully developed one, enabling routine, very precise, quasi-global measurements of sea level to be obtained. The elements of altimetric measurements are depicted in Figure 3.2.. For TOPEX/POSEIDON (T/P), the most precise mission, orbits are believed to be accurate to 2-3 cm, while the altimeter measurement system as a whole is accurate to 4 cm (see Fu et al., 1994). Goals for the follow-on mission, JASON, have been set at 1 cm level, or possibly better.
It is through discussion of this technique that the case for improved gravity field models can be most effectively made. Accuracy of the gravity field enters twice into discussion of Figure 3.2.. First, the predicted orbit of the satellite, i.e. its estimated height above the ocean, contains an error owing to uncertainties in knowledge of the gravity field at the satellite's altitude. Second, while sea surface height variations can easily be monitored by satellite altimetry, uncertainty in geoid height precludes the determination of absolute dynamic topography.

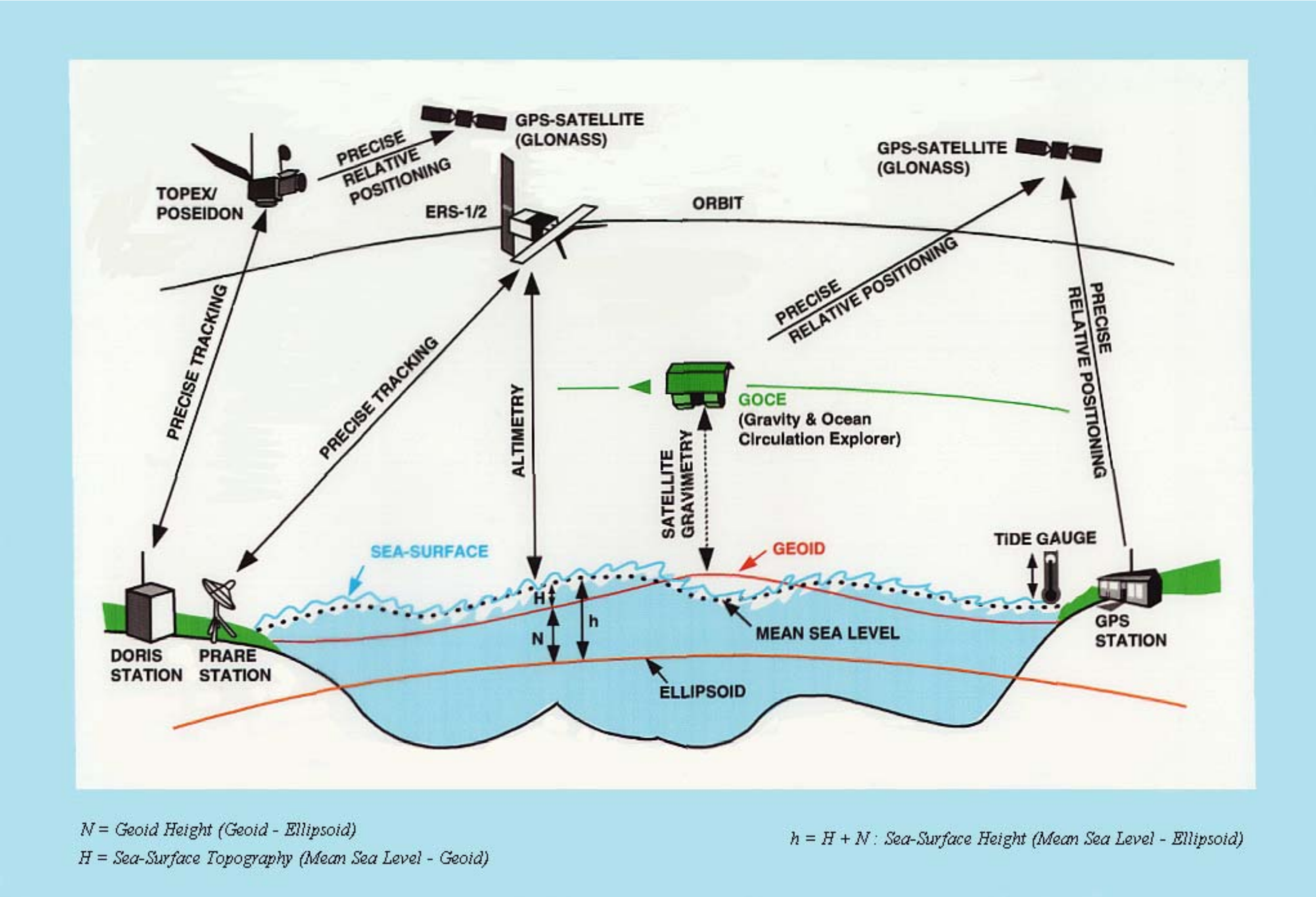


Figure 3.2. *Schematic of orbital tracking for satellite altimetry and for a GOCE type mission. Also shown is the separation of the mean sea surface into its component parts (ellipsoid, geoid and dynamic topography).*

3.4.3. Measuring the Absolute Ocean Circulation

Present Status

The study of the separation of the ocean dynamic topography from the geoid for the purpose of determining the global ocean circulation from altimeter data has a long history, dating at least from Wunsch and Gaposchkin (1980). Comparisons of T/P-derived dynamic topographies to hydrographic data can first be made directly. Figure 3.3. shows dynamic topography sections across the Indian and Pacific Oceans determined by hydrography (labelled 'Macdonald') and by T/P (where the JGM-2 geoid has been subtracted), as well as by the Semtner and Chervin model. The qualitative basin-scale correspondence can again be seen to be very encouraging, but a large signal in the T/P data is apparent at 185 E in the Pacific section. This fluctuation is almost certainly false, and at 20 S could correspond to an error of approximately 12 Sv in transport. It will be seen below that this level of transport error can be of significance in climate studies.

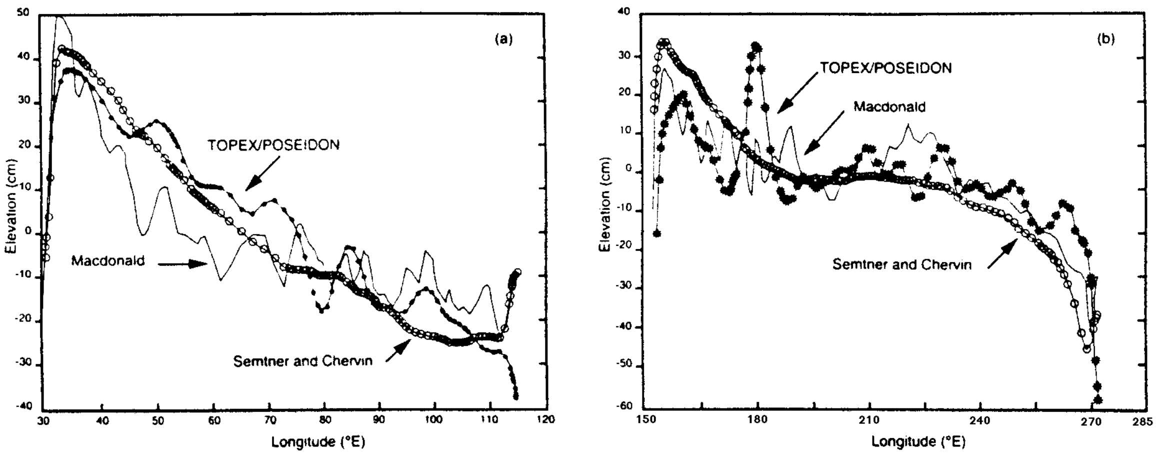


Figure 3.3. *Comparisons of dynamic topography obtained from a numerical ocean model, hydrography and T/P data for sections across the Indian Ocean. (C. Wunsch, private communication).*

Figure 3.4. (from Rapp et al., 1996) shows the difference between the two-year T/P dynamic topography (T/P sea level minus JGM-3 geoid) and the corresponding topography from a state-of-the-art numerical ocean model (Semtner and Chervin, 1992) to degree 14. Beyond degree 14, the spatial correlations between the two topographies are poor. The overall rms difference is 12 cm, with a maximum of about -60 cm. This is thought to result mostly from geoid errors and precludes a test of the quality of the model. Yet, in some areas, correspondence to models is found to be significantly better than to hydrographic data, for example in parts of the Antarctic Circumpolar Current area where about 40 per cent of the transport is barotropic and therefore not accessible to hydrographic measurements. This illustrates the point that absolute dynamic topography provides information on the ocean circulation, even on very large distance scales, which is difficult to get at by other means. The geoid precision as well as the ocean dynamic topography signals are a function of the spatial scales. Figure 3.5. (from Nerem et al., 1995) shows a spectral power comparison between the 'dynamic topography' and geoid model *error*. At long wavelengths (degree less than 14) the dynamic topography is separable from the geoid, at least for most areas of the world. However, geoid error becomes comparable to the dynamic topography signal around degree 14 in this example, that is at wavelength of about 3000 km. Furthermore, differences between four recent geoid models (OSU91A, JGM-2/OSU91A hybrid, JGM-2 and JGM-3) show significant and different undulations of this amplitude and larger than 1000 km wavelength.

In fact, the exact usefulness of an absolute dynamic topography to constrain the ocean circulation is difficult to assess, in addition to other oceanic data and knowledge of ocean dynamics in response to its atmospheric forcings: indeed, as stated above, the precision on the geoid varies with distance scale and the ocean signals on these various scales have different amplitudes; however the various scales of the ocean circulation are inter-related through ocean dynamics;

thus it may be that determining the mean absolute dynamic topography on large distance scales where the geoid is more precise is useful for constraining the whole circulation pattern. Following Wunsch and Gaposchkin (1980), several attempts have been made to merge all existing information on all distance scales (through inverse techniques) and to analyse whether existing surface dynamic topography data improve estimates of the ocean circulation and transports. The most recent attempt using the WOCE hydrographic sections and one of the most recent and precise geoids (JGM3 was here taken), is provided by Ganachaud et al. (1996). That analysis shows that the net heat fluxes across 20 of the WOCE sections are not modified significantly, nor their error reduced, by the addition of the dynamic topography information as known at present.

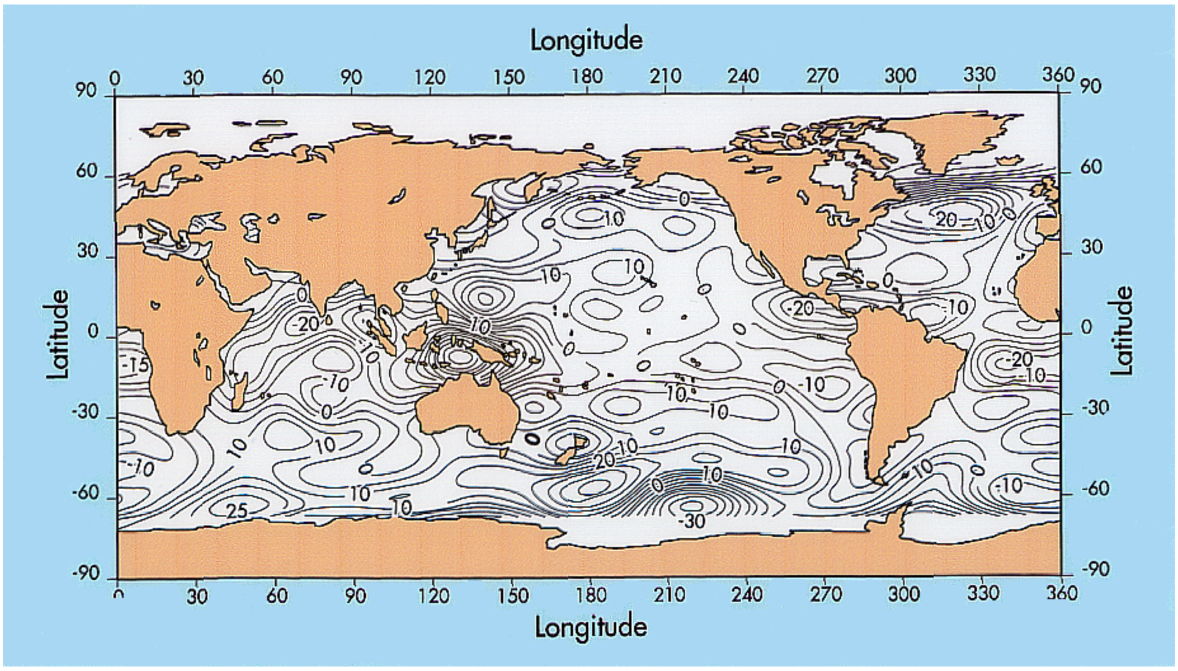


Figure 3.4. *Dynamic topography difference (2 year T/P minus JGM-3 surface, minus the corresponding surface from the Semptner-Chervin numerical model). Contour interval is 5 cm. From Rapp et al. (1996).*

Among other examples of what can be studied with finer resolution geoid models, Rapp and Wang (1994) have shown how realistic dynamic topography patterns can be obtained from precise altimetry and geoid information in regions where good local geoid models are available. Figure 3.6. describes a realistic circulation pattern for the Gulf Stream. Such circulation patterns provide a demonstration of the detail of information required globally, something which cannot be done without an advanced gravity mapping mission.

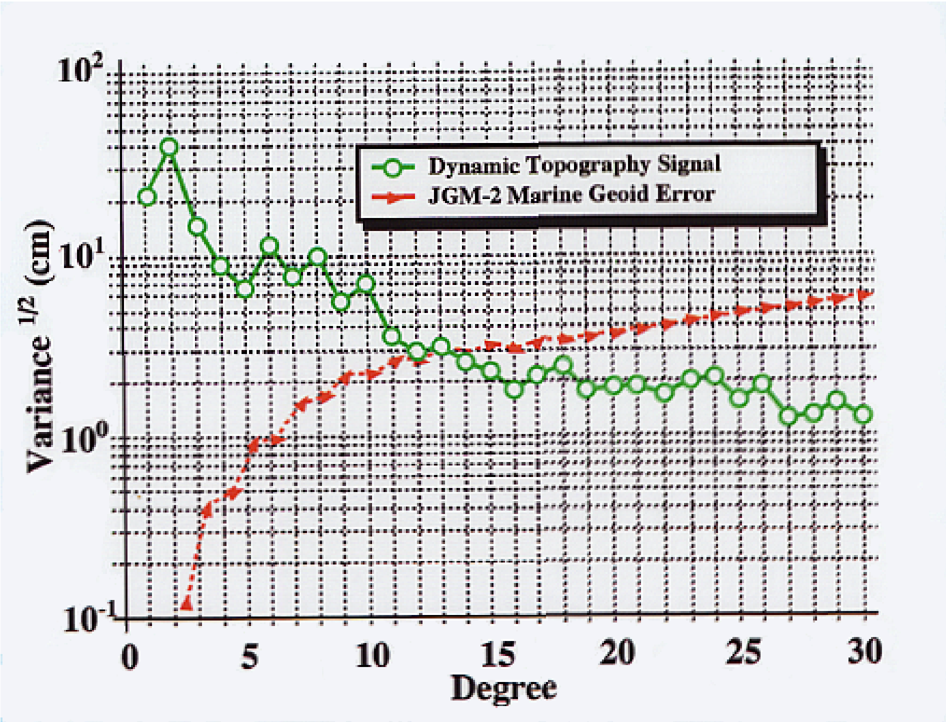


Figure 3.5. *Degree variance error spectrum of the geoid compared to the degree variance spectrum of the ocean dynamic topography. From Nerem et al., (1995).*

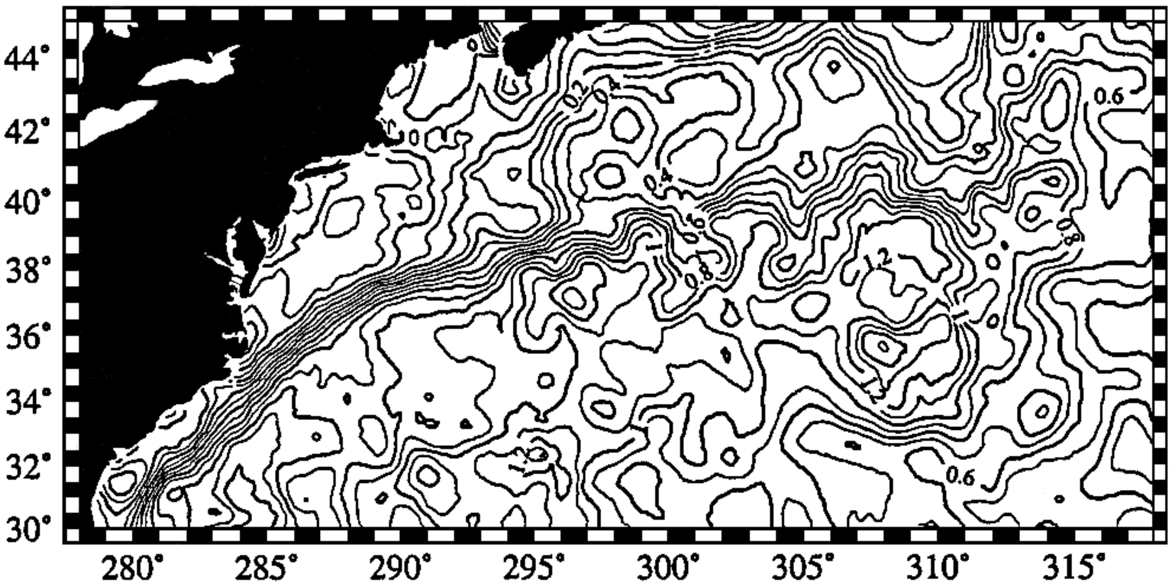


Figure 3.6. *Dynamic topography based on the separation between the mean sea surface and a local geoid in the western North Atlantic Ocean. Countour interval is 0.1 m. From Rapp and Wang (1994).*

Why Measure Mean Flows ?

With the T/P mission working so well with regard to sea level variability and to ocean circulation fluctuations, it is reasonable to ask why oceanographers need the mean circulation itself so much. There are several ways this question can be answered. First, it is through its mean flow that the ocean makes most of its transports of heat, fresh water or dissolved species and through which it controls the climate. Second, one knows that ocean currents tend to be unstable and to generate variabilities (e.g. eddies) which appear as sea surface height variations in altimetry. From numerical ocean models, one knows that the ocean can generate different degrees of variability depending on the mean flow and its interaction with other controlling factors (e.g., bathymetry). In return, through internal stresses, the variability acts as a brake or stimulant of the mean flows (e.g. in most western boundary currents, eddies act to speed up the mean flow). Both the mean flows and the eddies can carry heat, with one or the other being more important in particular locations. Thus, understanding the mean and understanding the variability must go hand-in-hand. Thirdly, one is dealing with non-linear processes. Consequently, if one is interested in studying transient perturbations of the system, then one should start from as good a description of it as possible. In that case, transient responses to forcing perturbations about that “mean” state have a better chance of description by means of linear parameterisation. That is essentially the motivation for the WOCE 'decadal snapshot' of the ocean, to provide a data set on which models might be based with potential for climate change prediction capability.

A particular issue is the understanding, as opposed to the monitoring, of global sea level change. For example, most of the projected rise of sea level over the next century is due to oceanic thermal expansion. The IPCC (1995) report shows that two of the main models used to simulate past and possible future changes in the IPCC study result in significant differences in predictions, in spite of the two models being superficially similar. In addition, General Circulation Model (GCM) simulations of potential future sea level change show a far from uniform response across the globe, owing to the consequent changes in ocean circulation implied from changes in climate forcings. Some studies suggest a larger than average rate of rise of sea level in the North Atlantic as a consequence of modification of the rate of bottom water formation (Mikolajewicz et al., 1990).

It is clear that more detailed understanding of the ocean circulation is required to refine the global average sea level change estimates of 13 - 111 cm in the next century. This range infers virtually no general impact at its low end to extensive impact to low-lying countries at the high end. Even the best estimate (49 cm) implies an order of magnitude increase in frequency of storm surge over-topping on the east coast of England. Most of this range of uncertainty can be assigned to lack of knowledge of ocean transports, including especially heat fluxes, and improvements in confidence of estimates is urgently required to enable effective coastal planning.

3.4.4. Requirements for measurement of the geoid for determining the ocean circulation

The oceanographic requirement from a space gravity mission, which would produce a geoid parameterisation with much improved accuracy down to spatial scales of several 100 km, was emphasised in the original T/P science plan (in 1981 !). It has been restated many times, most recently in two formal recommendations of the T/P Science Working Team (dated October 1990 and February 1993), by the European Group of Altimeter Specialists (Canaries Meeting, November 1995) and by the International Association for the Physical Sciences of the Ocean (IAPSO Executive Committee meeting, December 1995).

Requirements for measuring the dynamic topography

One first can describe the geoid requirement by describing the dynamic topography signal. Figure 3.7. shows the wavenumber spectra of the dynamic topography signals along various sections in the north Atlantic Ocean, deduced from a high resolution dynamical model adjusted to altimetric data (see Blayo et al., 1994). Constraining the mesoscale variability of the signal helps introduce the correct internal stress in the model and have the correct geometry of the mean signal. The figure thus shows realistic spectra for both the variable part of the signal (well known from altimetry) and the total (variability plus mean) signal. Of course, the energy of the signal is very different in a strong current (e.g. across the Gulf Stream) or middle of a gyre (e.g. middle of the Sargasso Sea). Everywhere, the energy of the total signal is larger than that of the variability signal at scales larger than about 500 km. Everywhere, the energy of the geoid signal (dashed line) is larger than that of the dynamic topography. In previous studies, requirements for spatial resolution of an improved geoid model have sometimes been expressed in terms of the wavelength implied by the Rossby radius of deformation. The Rossby radius is the smallest length scale over which geostrophic balance can occur. On smaller scales, the ocean circulation tends to get into turbulent cascades.

This is illustrated in Figure 3.7., where wavenumber spectra show a break near 500 km, with steep regular slopes to shorter wavelengths; the break corresponds to the scale of the geotrophic balance, while the steep slopes corresponds to the turbulent cascade. At mid-latitudes, the radius associated with the first baroclinic vertical mode (most currents at these latitudes have primarily baroclinic transports) is 10-40 km, increasing towards the equator. This translates into a requirement for precise knowledge of the geoid down to wavelengths of 60-250 km. The suggested spatial resolution for the mission is 60 to 250 km (although one might argue that such a spatial cut-off is slightly arbitrary, and one might consider that the requirement should contain a 'safety factor' of two or more). From detailed analysis of the variance preserving spectra in Figure 3.7., one can conclude that measuring the absolute dynamic topography everywhere, on distance scales of 250 km and more and at a precision of 10% of the signal, would require geoid precisions of order 2 cm on these scales.

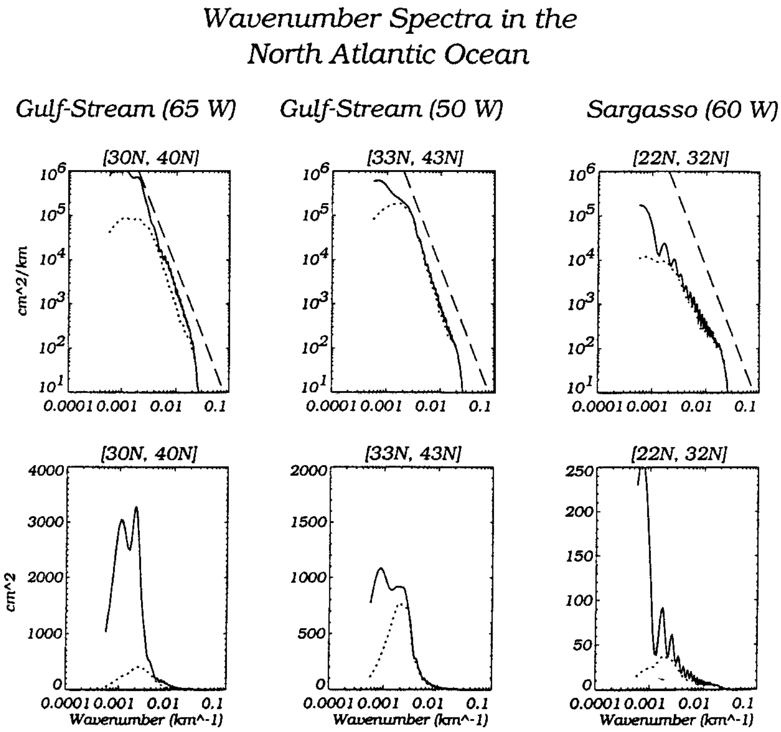


Figure 3.7. Comparison of the wavenumber spectra of the absolute dynamic topography (determined from a high resolution dynamical model adjusted to altimetric data; continuous lines), of the sea level variability spectra (determined from 2 years of T/P data; dotted lines) and of the geoid (determined from the altimetric data above the ellipsoid; dashed straight line) in various places of the Gulf Stream area. The Figure shows values across the jet (left), east of the New England seamounts (middle) and in the recirculation of the Sargasso sea (right). Spectra are given in variance preserving forms in the bottom panel. From Minster et al. (1996).

Thus, given the excellent precision of present altimetric data, measuring the absolute dynamic topography at a precision better than 10 % everywhere on scales of 250 km or more, requires a geoid of precision better than 2 cm on these scales.

Requirements for measurement of water and heat transports

The precision requirement stated above should also be qualified in terms of needs for studying oceanic quantities such as ocean transports. It is a feature of geostrophy that the distance scale on which the height error occurs does not matter for the resulting error in terms of water transport. Qualitatively, for a mean section of 5000 m at 30°N, a 7 Sv transport error arises from a 1 cm height (e.g. geoid) error. This is to be compared to formal baroclinic mass transport errors of approximately 2 Sv based on trans-ocean hydrographic sections (Macdonald, 1995). Macdonald's 2 Sv error is probably an underestimate by about a factor of 2 or 3; for example, her results are known to be affected particularly by systematic errors in the wind fields used to estimate Ekman fluxes. Nevertheless, this calculation emphasises that altimetric and geoid measurements must aim at 1 cm precision to have comparable errors to other oceanographic information.

Preliminary calculations have been made on how well meridional heat fluxes can be calculated. For example, C. Wunsch (private communication) provided a simple thought experiment which goes as follows. Imagine a zonal section at 30 N in 5000 m of water across the North Atlantic for which there is a cross-ocean integrated error of 1 cm in the working geoid model. The Atlantic is, say, 5000 km wide. If the absolute transport is calculated assuming barotropic flow, then an error of 7 Sv is obtained from the 1 cm geoid error. In any ocean model, the 7 Sv mis-modelling will have to be balanced by, for example, a corresponding error in the narrow western boundary current. If temperature anomalies across the section are typically 1 degree C, then heat flux errors of several x 10^{13} W result. Variations on this experiment can alter the heat flux error by an order of magnitude, but the amount is clearly of interest to climate modelling.

At present, model results have begun to suggest that the oceanic flux may be uncertain at first order (i.e near to 10^{15} W) owing to temporal fluctuations of the circulation. If that is the case, then measurement errors of 10^{13} W, or even 10^{14} W, would be of very great application.

Thus, at 30°N, even a 1 cm elevation error across a typical ocean basin (e.g. North Atlantic) corresponds to about 7 Sv error in water transport and approximately 10^{14} W in heat transports; these are large numbers but measurements to this accuracy would represent very significant improvements compared to present uncertainties. This sets the requirement for 1 cm precision over typically 1 000 km half-wavelength.

Results from inverse calculations

Flow scales are coupled through the details of the recirculation and through the equations of motion which govern the flow; information on all scales must thus be used simultaneously. Inverse calculations of the North Atlantic circulation have been performed using both in situ (hydrography and float data), dynamical constraints (geostrophy, Ekman pumping, mass conservation, conservation of potential vorticity at depth) and dynamic topography data using inverse techniques described by Mercier et al. (1993). Calculations have been made on a resolution of 500 km, compatible with the sampling by in situ data and with the expected resolution of future geoids. Figure 3.8. illustrates how the final error on the surface dynamic topography decreases when the variance of the geoid error is reduced from $(10\text{cm})^2$ (approximately representative of the present status), to $(5\text{cm})^2$ (say, the case for a gravity mission of intermediate class) and $(2\text{cm})^2$ (the GOCE case). In the first two cases, the a posteriori errors are large in places where few data are available or where ocean variability is large. The result confirms that improvement of the surface dynamic topography at the precision required above can only be reached through a GOCE type mission. Near the surface, precision on current velocities and transports are improved accordingly. The same analysis shows that errors on the current velocities at great depth (more precisely at 3000 dbars) were not significantly reduced by this surface improvement; as may be expected, the benefit of improved geoid errors reduces with depth.

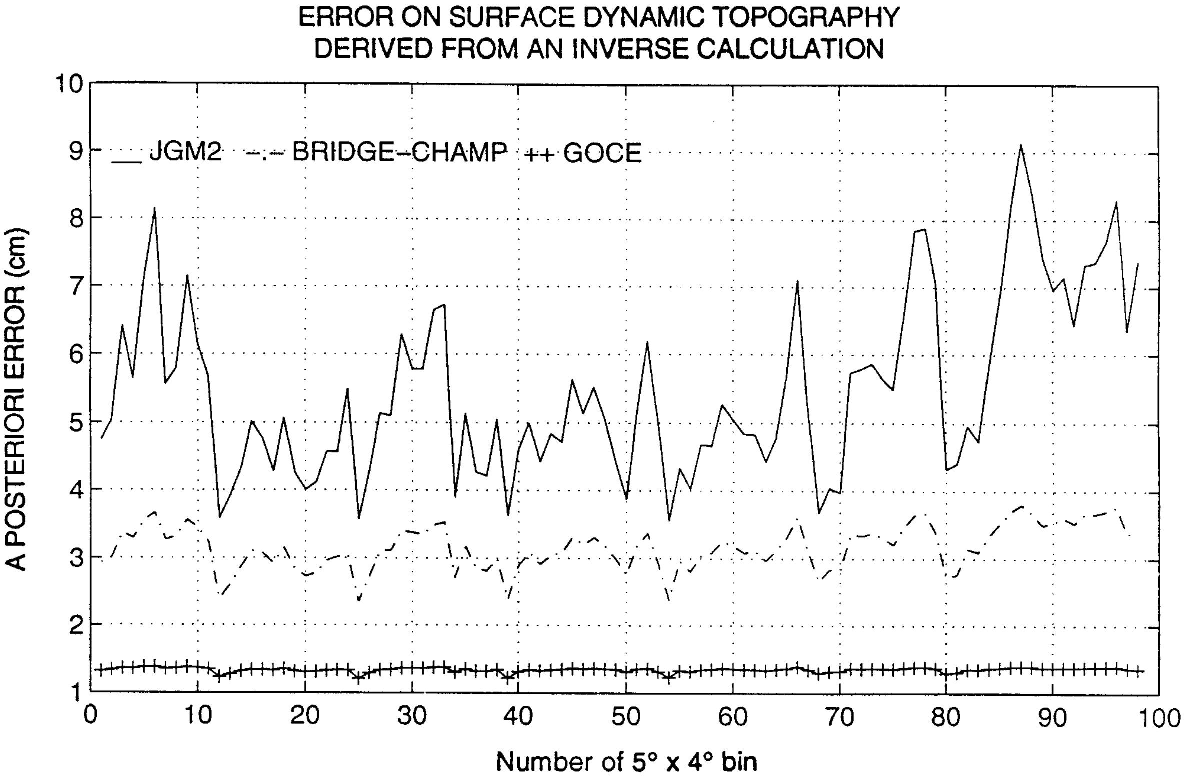


Figure 3.8. *A posteriori error on the surface dynamic topography as calculated using an inverse technique from in situ hydrography and float data and from a surface dynamic topography of error variance decreasing from (10 cm)^2 (named JGM2), to (5 cm)^2 (named BRIDGE-CHAMP), to (2 cm)^2, (named GOCE). The errors are given for all 5° x 4° boxes of the north Atlantic ocean between 5°N and 70°N.*

Thus, even when geoid information on all space scales and a priori knowledge from in situ data are used simultaneously, the error on ocean dynamic topography can be reduced at the necessary level only by a GOCE type mission; as information on the surface dynamic topography is related to currents in the water column, errors on currents and transports are then reduced in the first layers of the ocean but progressively less so with depth.

The best means to use this improved geoid information is through a dynamical circulation model. For example, at Hamburg, the circulation retrieval problem is being tackled using an adjoint model of the Hamburg Large Scale Geostrophic (LSG) global ocean circulation model.

We conclude that the future of oceanography will more and more rely on dynamical models and assimilation of all existing data. It is expected that it will be used globally as well as regionally and in routine, using near-real time altimetry data and realistic (high resolution) models by several groups in the world in about 5 to 10 years from now. Availability of accurate absolute dynamic topography data in this time frame will be of uttermost importance for the development of what can be called “operational oceanography”.

3.4.5. Tide gauges, sea level and the geoid

Similar statements related to inadequate present knowledge of the geoid can be made from a perspective of coastal tide gauge data. Tide gauges, together with GPS are monitoring straits as part of WOCE. Examples include sites spanning the Antarctic Circumpolar Current or ACC (Drake Passage, Amsterdam - Kerguelen, Australian 'choke point'), Gulf Stream locations (Bermuda - Charleston, Caribbean islands) and Straits of Gilbraltar. Where the distance across the strait is short, and where extensive local gravity information exists, then regional geoid models can be constructed enabling geoid-difference across the strait to be computed to good accuracy and to be employed together with geocentric sea surface height difference to determine absolute sea level gradients. The Straits of Dover are one example. However, as the distance increases, especially in areas where present gravity information is sparse, then geoid-difference becomes an imprecisely determined quantity. The ACC is a good example in this case. Distances across it are of the order of 1000-2000 km. ACC transport through the Drake Passage is considered to be approximately 130 Sv (mostly baroclinic) with a meridional change in dynamic topography of approximately 120 cm across the current. It has an rms variability at Drake Passage (mostly barotropic) of around 10 Sv on weekly timescales and longer, which translates into cross-Passage sea level variability of order 5 cm, which has been monitored by coastal and deep sea tide gauges for several years, and more recently by altimetry. If one asserts the need for knowing the mean transport as well as its rms variability (i.e. to within 5-10 Sv), then there is a requirement to know the geoid-difference to approximately 5 cm.

3.5. Contribution to atmospheric models

A drag measuring system (e.g. micro-accelerometer) must be installed on board any satellite of an advanced gravity mission, which will be in orbit below 500 km altitude (in fact at about 250-450 km of altitude for a GOCE or GRACE type mission). In some cases (e.g. GOCE) even a drag free spacecraft is envisaged. If the drag acting on the satellite can be determined to a precision of few percents, useful information can be obtained at these altitudes, to estimate the air total density and to better understand and predict the behaviour of the thermosphere. The total air density can be determined from the drag force measurements, provided that the other surface forces (direct solar radiation pressure, Earth’s albedo and infra-red pressure effects) are accurately modelled. Also the aerodynamic coefficients of the satellite must be known.

Many total density data sets have been obtained in the past. In the beginning, the data mainly consisted in the orbital decay information with a limited temporal resolution of a few days. However, from this kind of data as well as from other types of measurements obtained later, such as mass spectrometer and temperature, empirical models have been developed. They have been based on comparatively simple physical laws. It is of importance to emphasise how difficult it is to solve rigorously the physical equations particularly because no precise local energy budget is available. Local energy budget would oblige to monitor locally many physical parameters such as electro-magnetic radiation, emission and absorption rates, winds, energetic particle input, and in practice it is extremely difficult to do it. However, indicators of energy input such as the k_p geomagnetic index, the radio solar flux, allow to develop empirical models with a mean precision of the order of 15 to 20 %. Present data are available over few solar cycles only and since secular trends can occur, it is useful to monitor permanently the behaviour of several parameters of the thermosphere, such as the air total density. Therefore, although the spatial resolution might be limited within a band of few kilometers, and even if the spacecraft is flown at a single local time (e.g. heliosynchronous orbit), a gravity mission by-product should be an improvement of our knowledge of atmospheric physical parameters.

4. OBSERVATIONAL REQUIREMENTS

Although there is a real need for a global, accurate and high spatial resolution model of the Earth's gravity field and its related geoid heights, only gradual progress in achieving these goals has been made so far. Further significant progress is required if scientific aims are to be met. No dedicated space gravimetry mission has yet been flown, although data from many near-Earth space missions have been employed to improve the gravity field by assimilating their precise orbit information into gravity field solutions. The recent experience with TOPEX/POSEIDON data is an example. In spite of superb tracking of the satellite, its contribution to gravity field model improvement has been marginal because of its altitude. One concludes that there is no alternative to a low-flying gravity mission for significant progress in this area. We assume in the following that the elements of the mission are a high-accuracy Global Navigation Satellite System (GNSS) receiver and a gravity gradiometer (GOCE-type mission ; see section 2.8), on a satellite with low altitude in order to provide a gravity field signature compatible with the very high resolution of the two instruments. To ensure global completeness of the subsequent gravity model, which in fact impacts on its accuracy, a quasi-polar orbit is chosen.

4.1. Conversion of mission goals into observational requirements

The mission goals may be stated in terms of error estimates of each of the coefficients of the series of a spherical harmonic expansion (gravity model) or of the derived surface mean gravity or geoid values. The relationship between the observations and the estimated quantities may be obtained by error propagation from the known observation error characteristics (given as $E/\sqrt{Hz}$ or mgal). The mathematical relationship between the quantities is obtained using the property that the gravitational potential V is a harmonic function. And both measured and derived quantities are linear functionals applied on V. However the potential has infinite many components, so some side conditions have to be imposed in order to get from observations to result. Since the data are given in space, and the result is needed at the earth’s surface, errors are inflated when going down. This may be controlled by imposing restrictions on the magnitude of the result, called norm minimalisation or regularisation. The observations may have systematic errors. This may also be accounted for in simulation studies, and it has been shown that such errors may be removed by taking advantage of the fact that there are areas at the earth's surface where gravity is known precisely.

The different simulation studies differ mainly by the regularisation method used or by the quantities for which the error estimates are expressed. They all may use data at different altitudes and for different mission duration. A tremendous amount of work has been accomplished by a consortium of European teams with the support of the Agency in the recent years (CIGAR I, 1989; CIGAR II, 1990; CIGAR III, 1993 and 1995; CIGAR IV, 1996). All aspects of methodology, algorithms and software have been addressed, which results form a strong basis for the future use of the data from such a mission. From these studies we have been able to determine the observational requirements with a great deal of confidence.

4.1.2. Simulations using spherical harmonic coefficients

If V is supposed to be represented by a finite series in solid spherical harmonics of maximum degree L, then the result may be expressed as the standard deviation of $(L+1)^2$ coefficients. From these error estimates, errors on other quantities may be derived by linear error propagation. Several of such studies have been carried out, for instance by Colombo (1989), Schrama (1991), Balmino (1994). They all give the same result.

4.1.3. Boundary value approach

If the observations are interpolated to form a dense grid on a surface at satellite altitude, then there exists, for different data types, simple relationships between the coefficients of the spherical harmonic expansion of the data and that of V. Typically the coefficients must be multiplied by the degree l for an SST type measurement and by l^2 for an SGG measurement (this illustrates the amplification of errors as mentioned above). There is then a simple relationship between an idealised data set, and the coefficient errors, which is well suited to study e.g. the variation of errors as a function of altitude.

4.1.4. Covariance propagation approach

Numerical and statistical approximation methods may be used to work directly from the data (without gridding) to results on the earth's surface. They however generally require that systems of equations with full matrices are solved, and they have therefore primarily been used for regional simulations (see Arabelos & Tscherning, 1995). These studies have the advantage that the error may be directly propagated from the observations and that the actual gravity field variation may be taken into account.

4.2. Mission performance requirements

4.2.1. Required resolution and accuracy of the gravity field and geoid; a summary

The quantitative requirements coming from the arguments developed in sections 2 and 3 can be summarised in the next table. Taking advantage of numerous previous studies, we have kept only those phenomena for which the accuracy and resolution seem to be within the capability of a satellite mission based on the most advanced technology which is to-day at our reach.

	Accuracy		Spatial Resolution
	Geoid	Gravity	(half wavelength)
OCEAN CIRCULATION - Small scale - Basin scale	 2 cm <1 cm		 60-250 km 1 000 km
GEODYNAMICS - Continental lithosphere (thermal structure, post-glacial rebound) - Mantle composition, rheology - Ocean lithosphere and interaction with asthenosphere (subduction processes)		 1-2 mgals 1-2 mgals 5-10 mgals	 50-400 km 100-5000 km 100-200 km
GEODESY - ice and land vertical movements - rock basement under polar ice sheets - world-wide height system	 2 cm < 5 cm	 1-5 mgals	 100-200 km 50-100 km 50-100 km

4.2.2. Outline of gravity field and geoid recovery from space

Among the concepts proposed in section 2.6, we have retained the solution of a single spacecraft on a low orbit, and a combination of the gradiometry technique with high-low satellite to satellite tracking which benefits from existing satellite systems on high altitude orbits.

4.2.3. Derivation of instrument performance requirements via simulations and covariance analyses

The approach has been to perform simulations using a gravity potential representation by spherical harmonics (cf. 4.1.2.). These harmonics are recovered by inversion of the orbital perturbations in position (as monitored by high-low SST) and of gravity gradient measurements. To simplify matters and optimise the computational effort (for instance, about 40 000 coefficients must be estimated for a model truncated at degree 200), a linear model is assumed, the mean orbit of the satellite is taken circular and its plane precesses regularly around the Earth’s rotation axis so that, measurements being taken at fixed step interval, the resulting coverage is uniform (the orbit is also assumed to close itself after a very large number of revolutions, but this is not required in practice). In the linear model, the transfer functions between the harmonics of the potential and the orbital perturbations in the three directions: along-track (Δx), cross-track (Δy), radial (Δz), or the gravity gradient components (V_{xx} , V_{xy} , ...), carry the key to the determination of these harmonics. The expected a posteriori error variances of the harmonic coefficients are estimated, by a least squares scheme, from an a priori error model of observations. Band limited instruments can be considered and coloured noise can be introduced. In general the resulting system of equations may be unstable or even strictly singular (i.e. certain groups of coefficients cannot be determined). In such cases, one generally introduces some a priori or complementary information that permits a unique and stable determination of the solution. In the simulations reported below, a regularised solution is obtained by taking account of error variances of the coefficients for which good estimates are available (from a current long wavelength model), and signal variances as derived from a general model (Kaula's rule) for those coefficients for which no reasonable prior knowledge exists.

4.2.4. System performances vs. orbit parameters, instrumental errors, mission duration

A series of error propagation simulations was carried out for a GOCE-type mission (Rummel et al., 1995). For all simulation runs, realistical options were chosen in view of foreseen instrument characteristics and that would not lead to excessive complications. The parameters of the baseline simulation correspond to the proposed mission and are summarised in the next table.

Parameters of GOCE baseline simulation	
mission duration	8 months
orbit eccentricity	almost circular
orbit altitude	260 km
orbit inclination	96°.5 (sun-synchr.)
Residual errors in monitoring the high-low SST :	
Δx (along-track)	3 cm
Δy (cross-track)	2 cm
Δz (radial)	1 cm
gravity gradiometer :	
full diagonal instrument (V_{xx} , V_{yy} , V_{zz})	
error spectrum :	
. frequency $f < 10^{-3}\,Hz$	$1/f$
. $10^{-3}\,Hz < f < 10^{-1}\,Hz$	white noise = $5 \cdot 10^{-3}\,E/\sqrt{Hz}$
regularization with error degree variances of JGM-1 global model and signal degree variances of Kaula's rule.	
no omission error considered.	

For comparison a number of variants of the baseline simulation were carried out, by changing the white noise level of the gradiometer, or by changing the altitude, or by assuming that the radial component (V_{zz}) only was measured. It was found that :

- coefficients close to order zero are much less well determined, an effect caused by the polar gaps of a sun-synchronous orbit (without the high-low SST, this effect would be even more pronounced).
- the total gravity anomaly and geoid errors are dominated by the effect of the polar gaps.

Thus, the cumulative errors have to be computed as a function of colatitude. The results are displayed in figure 4.1..

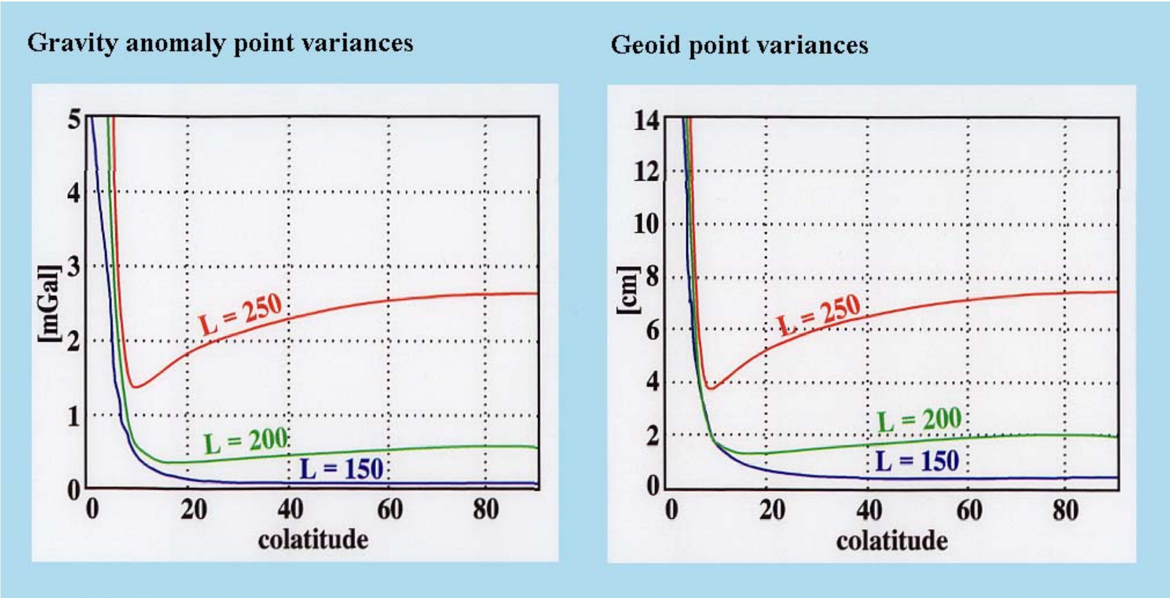


Figure 4.1. *Accumulated point error on geoid (a) and gravity anomaly (b), for series expansions up to degree and order L=150, L=200, L=250, as function of colatitude for the northern hemisphere (results are symmetrical with respect to equator), from the baseline simulation (parameters correspond to the proposed mission).*

From all the error simulations which were carried out, we draw the following conclusions :

- mission objectives are generally met for gravity anomalies, both in terms of accuracy and resolution
- mission objectives are met for geoid heights both in terms of accuracy and resolution limited to 100 km half-wavelength (*L* = 200)
- mission objectives are not met for areas close to the poles; this is due to the non-polar, sun-synchronous orbit
- high-low SST improves the low degree and order coefficients (particularly important for geoid determination) and minimises polar gap effect
- a small change in the measurement bandwidth of the gradiometer has no significant impact on the gravity recovery performance.
- spatial resolution (defined as degree where signal to noise ratio becomes one) is not sharply defined and depends on chosen prior signal degree variance model (e.g. Kaula); this implies that neither the maximum achievable degree nor the exact error contribution coming from the range of highest degree terms are well defined
- use of full diagonal gradiometer
  . determines angular velocities
  . provides redundancy
  . decorrelates geoid as well as gravity anomaly errors

- as a rule of thumb: in the interesting part of the gravity spectrum (say between degrees 60 and 150) improvement of gradiometer performance by one order of magnitude corresponds approximately to a satellite altitude decrease of 65 km.

4.3. Coverage and regularity

The simulation studies have shown that a mission duration of 8 months is adequate (for L = 250, minimum duration would be one month, uninterrupted and uniformly spaced over the globe; any extension of the mission duration by a factor of N reduces errors by a factor $\boxed{\quad}$, ideally). There is no need for a repeat orbit. Actually a constantly drifting ground track which ensures a uniform coverage is desirable. A polar orbit would certainly be ideal but the polar gap consequences can be diminished (see 4.5.3.).

4.4. Timeliness

Around year 2000-2002 we will have results from the two-frequency RA-2 radar altimeter on the ESA ENVISAT and also from the NASA-CNES TOPEX-POSEIDON follow on mission named JASON, as well as from the existing generation of altimeters. In order to fully exploit these new and highly precise data for climatic as well as new operational purposes an improved geoid is needed.

The present evolution of physical oceanography is towards near real-time use of altimeter data in operational ocean current modelling and prediction systems which should be ready by year 2000. Such systems are intended to help in climate as well as commercial applications (fisheries, coastal management, dispersion of pollution). It is crucial that geoid precision be improved to the level required for operational use of such data at a time compatible with the pace of development of the whole field. Also the related unification of height systems contribute to these goals.

4.5. Ground observations and support

4.5.1. IGS in support of SST

The International GPS Service for Geodynamics is an international service which provides precise orbits for the GPS satellites and station coordinates for more than 100 tracking stations. Its services are provided free of charge for scientific purposes, and the results are expected to be accessible for users also in the next decade.

4.5.2. Ground based gravity data for calibration

There are some areas on the continents where the gravity field has been surveyed in much detail, and where the gravity field is very smooth. In such areas the gravity information may be up-ward continued with high precision to satellite altitude, and used to calculate the gravity vector and the gravity gradient components. Error-propagation studies show that this may be done with an error of 0.02 E for the V_{zz} component at 200 km altitude. Such areas may hence be used to calibrate or verify the results obtained by the satellite gradiometer.

4.5.3. Coverage of polar gaps

For a quasi polar orbit, e.g. a sun-synchronous orbit, a small area at the poles will not be covered by the satellite tracks, impacting the accuracy of the recovered gravity field and geoid over this specific but small area as shown above (cf. 4.2.4.). These polar gaps (latitudes above + 82° or below - 82°) are nearly void of gravity data. An effort is however expected to be made in connection with new ice-core drillings in Antarctica, and for the Arctic area airborne gravity may be collected in the coming years.

Fortunately, such a lack of data would not degrade the use of the mission results for the main applications, since geoid height differences between points at latitudes below 82 degrees may be calculated with a very good precision, see Figure 4.1..

4.6. Conclusion

The simulation studies have resulted in the translation of scientific goals into system and observational requirements. Figures 4.2. and 4.3. summarise, as a function of the spatial resolution, the scientific requirements in terms of gravity and geoid accuracy and the capabilities of a GOCE-type mission. They clearly demonstrate that such a mission would be a giant step for geophysics in general.

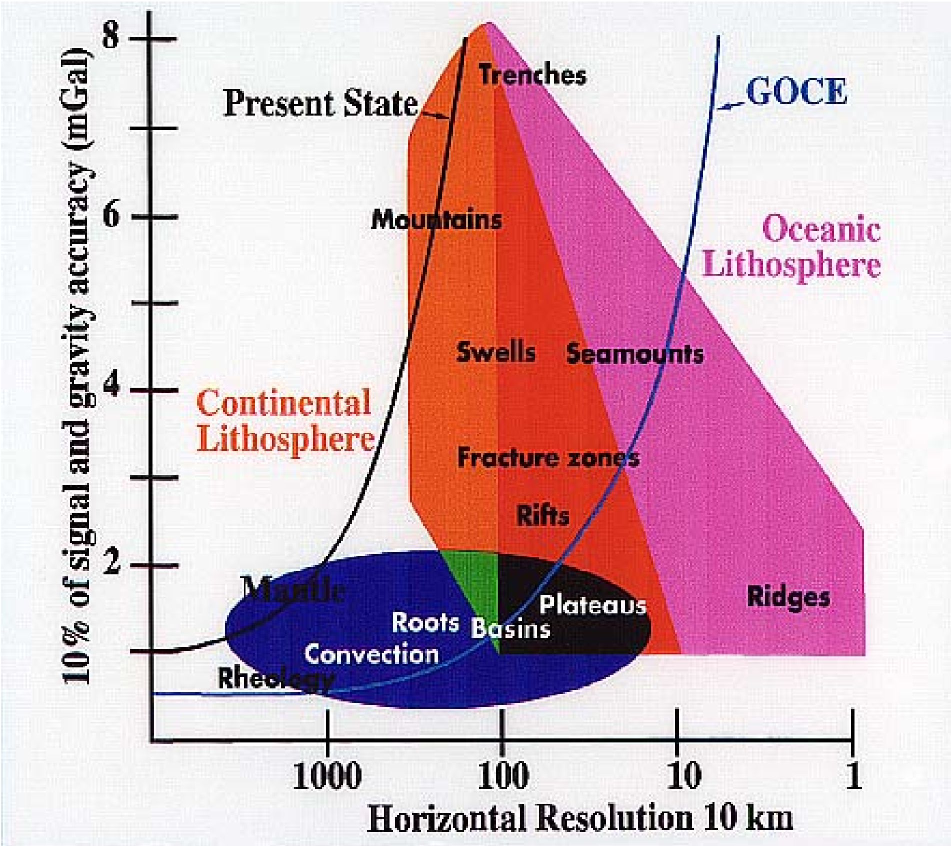


Figure 4.2. *Schematic description of the required accuracy (estimated at approximately 10 per cent of the gravity signal) as a function of horizontal resolution necessary to resolve the quoted geodynamical and tectonic features. The two curves represent the present and potential accuracy after a GOCE-type mission.*

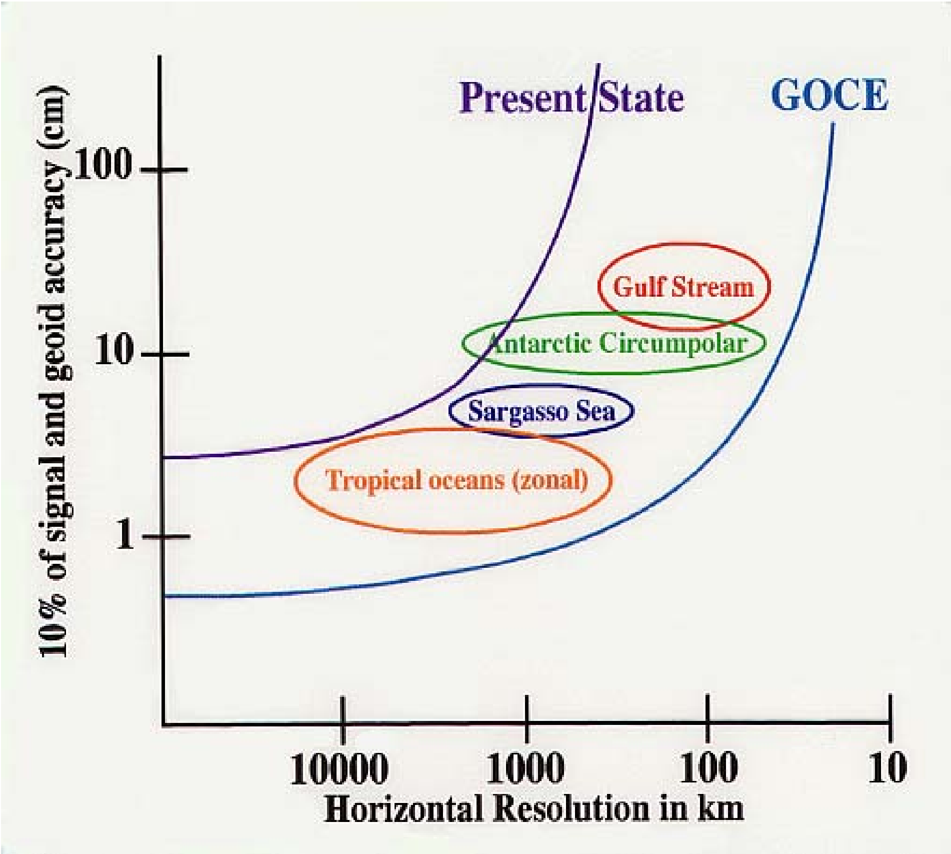


Figure 4.3. *Schematic description of 10 % of the dynamic topography signals of ocean circulation features (10 % representing order of magnitude knowledge of the mean flows, and of the variability) and geoid accuracy as a function of spatial scales obtainable from present models and after a GOCE-type mission.*

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List of Acronyms

ACC	Antarctic Circumpolar Current
BGI	Bureau Gravimétrique International
BRIDGE (France)	Mini-Satellite Concept to Bridge the Past and Future in Gravity Field Research
CHAMP	Catastrophes and Hazard Monitoring and Prediction (FRG - France - USA)
DORIS	Doppler Orbitography and Radiopositioning Integrated by Satellite (France)
CIGAR	Consortium for Investigations in Gravity Anomalies Recovery
CLIVAR	Climate Variability Programme
CNES	Centre National d'Etudes Spatiales (France)
EGM	Earth Gravity Model
ERS	European Remote Sensing satellite (ESA)
ESA	European Space Agency
GCM	General Circulation Model
GFO	Geosat Follow On (USA)
GLONASS	Global Navigation Satellite System (Russia)
GNSS	Global Navigation Satellite System (based on GPS and GLONASS)
GOCE	Gravity Field and Steady-State Ocean Circulation Explorer
GOOS	Global Ocean Observing System
GPS	Global Positioning System (USA)
GRM	Gravity Research Mission (USA)
IGS	International GPS Geodynamics Service
IPCC	International Panel on Climate Change
JASON	Topex-Poseidon follow-on mission (USA- France)
JGM	Joint Gravity Model (USA - France)
LEO	Low Earth Orbiter
LSG	Large Scale Geostrophic (LSG)
NASA	National Aeronautics and Space Administration (USA)
NIMA USA)	National Imagery and Mapping Agency (ex-DMA : **D**efense **M**apping **A**gency ;
OSU	Ohio State University (USA)
PGR	Post-Glacial Rebound
RA-2	Radar Altimeter (on-board Envisat)
SAR	Synthetic Aperture Radar
SGG	Satellite Gravity Gradiometry
SST	Satellite to Satellite Tracking
STEP	Satellite Test of Equivalence Principle
T/P	TOPEX/POSEIDON (USA - France)
WCRP	World Climate Research Programme
WOCE	World Ocean Circulation Experimen