ASSOCIATION INTERNATIONALE DE GÉODÉSIE

BUREAU

GRAVIMETRIQUE

INTERNATIONAL

BULLETIN D'INFORMATION

N° 90

Juillet 2002

18, Avenue Édouard Belin
31401 TOULOUSE CEDEX 4
FRANCE

INFORMATIONS for CONTRIBUTORS

Contributors should follow as closely as possible the rules below :

Manuscripts should be typed (single spaced), on one side of plain paper 21 cm x 29,7 cm with a 2 cm margin on the left and right hand sides as well as on the bottom, and with a 3 cm margin at the top (as indicated by the frame drawn on this page).

NOTA : ***The publisher welcomes the manuscripts which have been prepared using WORD 6 for Macintosh and also accepts ASCII files on diskettes 3"5.***

Title of paper. Titles should be carefully worded to include only key words.

Abstract. The abstract of a paper should be informative rather than descriptive. It is not a table of contents. The abstract should be suitable for separate publication and should include all words useful for indexing. Its length should be limited to one typescript page.

Footnotes. Because footnotes are distracting, they should be avoided as much as possible.

Mathematics. For papers with complicated notation, a list of symbols and their definitions should be provided as an appendix. Symbols that must be handwritten should be identified by notes in the margin. Ample space (1.9 cm above and below) should be allowed around equations so that type can be marked for the printer. Where an accent or underscore has been used to designate a special type face (e.g., boldface for vectors, script for transforms, sans serif for tensors), the type should be specified by a note in a margin. Bars cannot be set over superscripts or extended over more than one character. Therefore angle brackets are preferable to accents over characters. Care should be taken to distinguish between the letter O and zero, the letter l and the number one, kappa and k, mu and the letter u, nu and v, eta and n, also subscripts and superscripts should be clearly noted and easily distinguished. Unusual symbols should be avoided.

Acknowledgements. Only significant contributions by professional colleagues, financial support, or institutional sponsorship should be included in acknowledgements.

References. A complete and accurate list of references is of major importance in review papers. All listed references should be cited in text. A complete reference to a periodical gives author (s), title of article, name of journal, volume number, initial and final page numbers (or statement "in press"), and year published. A reference to an article in a book, pages cited, publisher's location, and year published. When a paper presented at a meeting is referenced, the location, dates, and sponsor of the meeting should be given. References to foreign works should indicate whether the original or a translation is cited. Unpublished communications can be referred to in text but should not be listed. Page numbers should be included in reference citations following direct quotations in text. If the same information have been published in more than one place, give the most accessible reference ; e.g. a textbook is preferable to a journal, a journal is preferable to a technical report.

Table. Tables are numbered serially with Arabic numerals, in the order of their citation in text. Each table should have a title, and each column, including the first, should have a heading. Column headings should be arranged to that their relation to the data is clear.

Footnotes for the tables should appear below the final double rule and should be indicated by a, b, c, etc. Each table should be arranged to that their relation to the data is clear.

Illustrations. Original drawings of sharply focused glossy prints should be supplied, with two clear Xerox copies of each for the reviewers. Maximum size for figure copy is (25.4 x 40.6 cm). After reduction to printed page size, the smallest lettering or symbol on a figure should not be less than 0.1 cm high ; the largest should not exceed 0.3 cm. All figures should be cited in text and numbered in the order of citation. Figure legends should be submitted together on one or more sheets, not separately with the figures.

Mailing. Typescripts should be packaged in stout padded or stiff containers ; figure copy should be protected with stiff cardboard.

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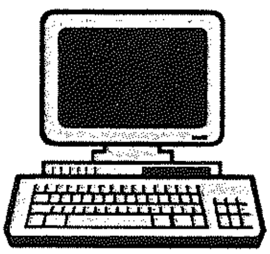

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BUREAU GRAVIMÉTRIQUE INTERNATIONAL

Toulouse

BULLETIN D'INFORMATION

Juillet 2002

N° 90

Publié pour le Conseil International des
Unions Scientifiques avec l'aide financière
de l'UNESCO
Subvention UNESCO 2000 DG/2.1/414/50

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PART I

INTERNAL MATTERS

GENERAL INFORMATION

1. HOW TO OBTAIN THE BULLETIN
2. HOW TO REQUEST DATA
3. USUAL SERVICES B.G.I. CAN PROVIDE
4. PROVIDING DATA TO B.G.I.

1. HOW TO OBTAIN THE BULLETIN

The Bulletin d'Information of the Bureau Gravimétrique International is issued twice a year, generally at the end of June and end of December.

The Bulletin contains general information on the community, on the Bureau itself. It informs about the data available, about new data sets...

It also contains contributing papers in the field of gravimetry, which are of technical character. More scientifically oriented contributions should better be submitted to appropriate existing journals.

Communications presented at general meeting, workshops, symposia, dealing with gravimetry (e.g. IGC, S.S.G.'s,...) are published in the Bulletin when appropriate - at least by abstract.

Once every four years, an issue contains the National Reports as presented at the International Gravity Commission meeting. Special issues may also appear (once every two years) which contain the full catalogue of the holdings.

About three hundred individuals and institutions presently receive the Bulletin.

You may :

- either request a given bulletin, by its number (90 have been issued as of July 31, 2002 but numbers 2,16, 18,19 are out of print).

- or subscribe for regularly receiving the two bulletins per year (the special issues are obtained at additional cost).

Requests should be sent to:

Mrs. Nicole LESTIEU
CNES/BGI
18, Avenue Edouard Belin
31401 TOULOUSE CEDEX 4 - FRANCE

Bulletins are sent on an exchange basis (free of charge) to individuals, institutions which currently provide informations, data to the Bureau. For other cases, the price of each issue is 75 FF.

2. HOW TO REQUEST DATA

2.1. Stations descriptions Diagrams for Reference, Base Stations (including IGSN 71's)

Request them by number, area, country, city name or any combination of these.

When we have no diagram for a given request, but have the knowledge that it exists in another center, we shall in most cases forward the request to this center or/and tell the inquiring person to contact the center.

Do not wait until the last moment (e.g. when you depart for a cruise) for asking us the information you need: station diagrams can only reach you by mail, in many cases.

2.2. G-Value at Base Stations

Treated as above.

2.3. Mean Anomalies, Mean Geoid Heights, Mean Values of Topography

The geographic area must be specified (polygon). According to the data set required, the request may be forwarded in some cases to the agency which computed the set.

2.4. Gravity Maps

Request them by number (from the catalogue), area, country, type (free-air, Bouguer...), scale, author, or any combination of these.

Whenever available in stock, copies will be sent without extra charges (with respect to usual cost - see § 3.3.2.). If not, two procedures can be used:

- we can make (poor quality) black and white (or ozalide-type) copies at low cost,

- color copies can be made (at high cost) if the user wishes so (after we obtain the authorization of the editor).

The cost will depend on the map, type of work, size, etc... In both cases, the user will also be asked to send his request to the editor of the map before we proceed to copying.

2.5. Gravity Measurements

2.5.1. CD-Roms

The non confidential data, which have been validated by various procedures are available on two CD-ROMs.

The price of these is :

- 800 (Eight hundred) French francs for individual scientists, universities and research laboratories or groups working in geodesy or geophysics.

- 3000 (Three thousand) French francs for all other users.

Most essential quantities are given, in a compressed format. The package includes a user's guide and software to retrieve data according to the area, the source code, the country.

2.5.2. Data stored in the general data base

BGI is now using the ORACLE Data Base Management System. One implication is that data are stored in only one format (though different for land and marine data), and that archive files do not exist anymore.

There are two distinct formats for land or sea gravity data, respectively EOL and EOS.

EOL
LAND DATA FORMAT
RECORD DESCRIPTION
126 characters

Col. 1-8 B.G.I. source number (8 char.)

9-16 Latitude (unit : 0.00001 degree) (8 char.)

17-25 Longitude (unit : 0.00001 degree) (9 char.)

26-27 Accuracy of position (2 char.)

The site of the gravity measurements is defined in a circle of radius R

0 = no information
1 - R <= 5 Meters
2 = 5 < R <= 20 M (approximately 0'01)
3 = 20 < R <= 100 M
4 = 100 < R <= 200 M (approximately 0'1)
5 = 200 < R <= 500 M
6 = 500 < R <= 1000 M
7 = 1000 < R <= 2000 M (approximately 1')
8 = 2000 < R <= 5000 M
9 = 5000 M < R
10...

28-29 System of positioning (2 char.)

0 = no information
1 = topographical map
2 = trigonometric positioning
3 = satellite

30 Type of observation (1 char.)

1 = current observation of detail or other observations of a 3rd or 4th order network
2 = observation of a 2nd order national network
3 = observation of a 1st order national network
4 = observation being part of a nation calibration line
5 = coastal ordinary observation (Harbour, Bay, Sea-side...)
6 = harbour base station

31-38 Elevation of the station (unit : centimeter) (8 char.)

39-40 Elevation type (2 char.)

1 = Land
2 = Subsurface
3 = Lake surface (above sea level)
4 = Lake bottom (above sea level)
5 = Lake bottom (below sea level)
6 = Lake surface (above sea level with lake bottom below sea level)
7 = Lake surface (below sea level)
8 = Lake bottom (surface below sea level)
9 = Ice cap (bottom below sea level)
10 = Ice cap (bottom above sea level)
11 = Ice cap (no information about ice thickness)

41-42 Accuracy of elevation (2 char.)

0 = no information
1 = E <= 0.02 M
2 = .02 < E <= 0.1 M
3 = .1 < E <= 1
4 = 1 < E <= 2
5 = 2 < E <= 5
6 = 5 < E <= 10
7 = 10 < E <= 20
8 = 20 < E <= 50
9 = 50 < E <= 100
10 = E superior to 100 M

43-44	Determination of the elevation	(2 char.)
	0 = no information	
	1 = geometrical levelling (bench mark)	
	2 = barometrical levelling	
	3 = trigonometric levelling	
	4 = data obtained from topographical map	
	5 = data directly appreciated from the mean sea level	
	6 = data measured by the depression of the horizon	
	7 = satellite	
45-52	Supplemental elevation (unit : centimeter)	(8 char.)
53-61	**Observed gravity** (unit : microgal)	(9 char.)
62-67	**Free air anomaly** (0.01 mgal)	(6 char.)
68-73	**Bouguer anomaly** (0.01 mgal)	(6 char.)
	Simple Bouguer anomaly with a mean density of 2.67. No terrain correction	
74-76	Estimation standard deviation free-air anomaly (0.1 mgal)	(3 char.)
77-79	Estimation standard deviation bouguer anomaly (0.1 mgal)	(3 char.)
80-85	**Terrain correction** (0.01 mgal)	(6 char.)
	computed according to the next mentioned radius & density	
86-87	Information about terrain correction	(2 char.)
	0 = no topographic correction	
	1 = tc computed for a radius of 5 km (zone H)	
	2 = tc computed for a radius of 30 km (zone L)	
	3 = tc computed for a radius of 100 km (zone N)	
	4 = tc computed for a radius of 167 km (zone 02)	
	11 = tc computed from 1 km to 167 km	
	12 = tc computed from 2.3 km to 167 km	
	13 = tc computed from 5.2 km to 167 km	
	14 =tc (unknown radius)	
	15 = tc computed to zone M (58.8 km)	
	16 = tc computed to zone G (3.5 km)	
	17 = tc computed to zone K (18.8 km)	
	25 = tc computed to 48.6 km on a curved Earth	
	26 = tc computed to 64. km on a curved Earth	
88-91	Density used for terrain correction	(4 char.)
92-93	Accuracy of gravity	(2 char.)
	0 = no information	
	1 = E <= 0.01 mgal	
	2 = .01 < E <= 0.05 mgal	
	3 = .05 < E <= 0.1 mgal	
	4 = 0.1 < E <= 0.5 mgal	
	5 = 0.5 < E <= 1. mgal	
	6 = 1. < E <= 3. mgal	
	7 = 3. < E <= 5. mgal	
	8 = 5. < E <= 10 mgal	
	9 = 10. < E <= 15. mgal	
	10 = 15. < E <= 20. mgal	
	11 = 20. < E mgal	
94-99	Correction of observed gravity (unit : microgal)	(6 char.)
100-105	**Reference station**	(6 char.)
	This station is the base station (BGI number) to which the concerned station is referred	

106-108 Apparatus used for the measurement of G (3 char.)

0.. no information
1.. pendulum apparatus before 1960
2.. latest pendulum apparatus (after 1960)
3.. gravimeters for ground measurements in which the variations of G are equilibrated of detected using the following methods :
30 = torsion balance (Thyssen...)
31 = elastic rod
32 = bifilar system
34 = Boliden (Sweden)
4.. Metal spring gravimeters for ground measurements
41 = Frost
42 = Askania (GS-4-9-11-12), Graf
43 = Gulf, Hoyt (helical spring)
44 = North American
45 = Western
47 = Lacoste-Romberg
48 = Lacoste-Romberg, Model D (microgravimeter)
5.. Quartz spring gravimeter for ground measurements
51 = Norgaard
52 = GAE-3
53 = Worden ordinary
54 = Worden (additional thermostat)
55 = Worden worldwide
56 = Cak
57 = Canadian gravity meter, sharpe
58 = GAG-2
59 = SCINTREX CG2
6.. Gravimeters for under water measurements (at the bottom of the sea or of a lake)
60 = Gulf
62 = Western
63 = North American
64 = Lacoste-Romberg

109-111 **Country code** (BGI) (3 char.)

112 **Confidentiality** (1 char.)

0 = without restriction
.....1 = with authorization
2 = classified

113 **Validity** (1 char.)

0 = no validation
1 = good
2 = doubtful
3 = lapsed

114-120 Numbering of the station (original) (7 char.)

121-126 Sequence number (6 char.)

EOS
SEA DATA FORMAT
RECORD DESCRIPTION
146 characters

Col. 1-8 **B.G.I.** source number (8 char.)
9-16 **Latitude** (unit : 0.00001 degree) (8 char.)
17-25 **Longitude** (unit : 0.00001 degree) (9 char.)
26-27 Accuracy of position (2 char.)
The site of the gravity measurements is defined in a circle of radius R
0 = no information
1 - R <= 5 Meters
2 = 5 < R <= 20 M (approximately 0'01)
3 = 20 < R <= 100 M
4 = 100 < R <= 200 M (approximately 0'1)
5 = 200 < R <= 500 M
6 = 500 < R <= 1000 M
7 = 1000 < R <= 2000 M (approximately 1')
8 = 2000 < R <= 5000 M
9 = 5000 M < R
10...

28-29 System of positioning (2 char.)
0 = no information
1 = Decca
2 = visual observation
3 = radar
4 = loran A
5 = loran C
6 = omega or VLF
7 = satellite
8 = solar/stellar (with sextant)

30 Type of observation (1 char.)
1 = individual observation at sea
2 = mean observation at sea obtained from a continuous recording

31-38 **Elevation of the station** (unit : centimeter) (8 char.)
39-40 Elevation type (2 char.)
1 = ocean surface
2 = ocean submerged
3 = ocean bottom

41-42 Accuracy of elevation (2 char.)
0 = no information
1 = E <= 0.02 Meter
2 = .02 < E <= 0.1 M
3 = .1 < E <= 1
4 = 1 < E <= 2
5 = 2 < E <= 5
6 = 5 < E <= 10
7 = 10 < E <= 20
8 = 20 < E <= 50
9 = 50 < E <= 100
10 = E superior to 100 Meters

43-44 Determination of the elevation (2 char.)
0 = no information
1 = depth obtained with a cable (meters)
2 = manometer depth
3 = corrected acoustic depth (corrected from Mathew's tables, 1939)
4 = acoustic depth without correction obtained with sound speed 1500 M/sec. (or 820 fathom/sec)
5 = acoustic depth obtained with sound speed 1463 M/sec (800 fathom/sec)
6 = depth interpolated on a magnetic record
7 = depth interpolated on a chart

45-52 Supplemental elevation (8 char.)
53-61 **Observed gravity** (unit : microgal) (9 char.)
62-67 **Free air anomaly** (0.01 mgal) (6 char.)

68-73	**Bouguer anomaly** (0.01 mgal)	(6 char.)
	Simple Bouguer anomaly with a mean density of 2.67. No terrain correction	
74-76	Estimation standard deviation free-air anomaly (0.1 mgal)	(3 char.)
77-79	Estimation standard deviation bouguer anomaly (0.1 mgal)	(3 char.)
80-85	**Terrain correction** (0.01 mgal)	(6 char.)
	computed according to the next mentioned radius & density	
86-87	Information about terrain correction	(2 char.)
	0 = no topographic correction	
	1 = tc computed for a radius of 5 km (zone H)	
	2 = tc computed for a radius of 30 km (zone L)	
	3 = tc computed for a radius of 100 km (zone N)	
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	14 =tc (unknown radius)	
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	16 = tc computed to zone G (3.5 km)	
	17 = tc computed to zone K (18.8 km)	
	25 = tc computed to 48.6 km on a curved Earth	
	26 = tc computed to 64. km on a curved Earth	
88-91	Density used for terrain correction	(4 char.)
92-93	Mathew's zone	(2 char.)
	when the depth is not corrected depth, this information is necessary. For example : zone 50	
	for the Eastern Mediterranean Sea	
94-95	Accuracy of gravity	(2 char.)
	0 = no information	
	1 = E <= 0.01 mgal	
	2 = .01 < E <= 0.05 mgal	
	3 = .05 < E <= 0.1 mgal	
	4 = 0.1 < E <= 0.5 mgal	
	5 = 0.5 < E <= 1. mgal	
	6 = 1. < E <= 3. mgal	
	7 = 3. < E <= 5. mgal	
	8 = 5. < E <= 10. mgal	
	9 = 10. < E <= 15. mgal	
	10 = 15 < E <= 20. mgal	
	11 = 20. < E mgal	
96-101	Correction of observed gravity (unit : microgal)	(6 char.)
102-110	Date of observation	(9 char.)
	in Julian day - 2 400 000 (unit : 1/10 000 of day)	
111-113	Velocity of the ship (0.1 knot)	(3 char.)
114-118	Eötvös correction (0.1 mgal)	(5 char.)
119-121	**Country code** (BGI)	(3 char.)
122	**Confidentiality**	(1 char.)
	0 = without restriction	
	1 = with authorization	
	2 = classified	
123	**Validity**	(1 char.)
	0 = no validation	
	1 = good	
	2 = doubtful	
	3 = lapsed	
124-130	Numbering of the station (original)	(7 char.)
131-136	**Sequence number**	(6 char.)
137-139	**Leg number**	(3 char.)
140-145	**Reference station**	(6 char.)

Whenever given, the theoretical gravity (γ_o), free-air anomaly (FA), Bouguer anomaly (BO) are computed in the 1967 geodetic reference system.

The approximation of the closed form of the 1967 gravity formula is used for theoretical gravity at sea level:

$$\gamma_o = 978031.85 * [1 + 0.005278895 * \sin^2(\phi) + 0.000023462 * \sin^4(\phi)], \text{ mgals}$$

where ϕ is the geographic latitude.

The formulas used in computing FA and BO are summarized below.

Formulas used in computing free-air and Bouguer anomalies

Symbols used :

g	*: observed value of gravity*
γ	*: theoretical value of gravity (on the ellipsoid)*
Γ	*: vertical gradient of gravity (approximated by 0.3086 mgal/meter)*
H	*: elevation of the physical surface of the land, lake or glacier (H = o at sea surface), positive upward*
D_1	*: depth of water, or ice, positive downward*
D_2	*: depth of a gravimeter measuring in a mine, in a lake, or in an ocean, counted from the surface, positive downward*
G	*: gravitational constant ($667.2\ 10^{-13}\ m^3\ kg^{-1}\ s^{-2}$) $\Rightarrow k = 2\pi G$*
ρ_c	*: mean density of the Earth's crust (taken as 2670 kg m^{-3})*
ρ_w^f	*: density of fresh water (1000 kg m^{-3})*
ρ_w^s	*: density of salted water (1027 kg m^{-3})*
ρ_i	*: density of ice (917 kg m^{-3})*
FA	*: free-air anomaly*
BO	*: Bouguer anomaly*

Formulas :

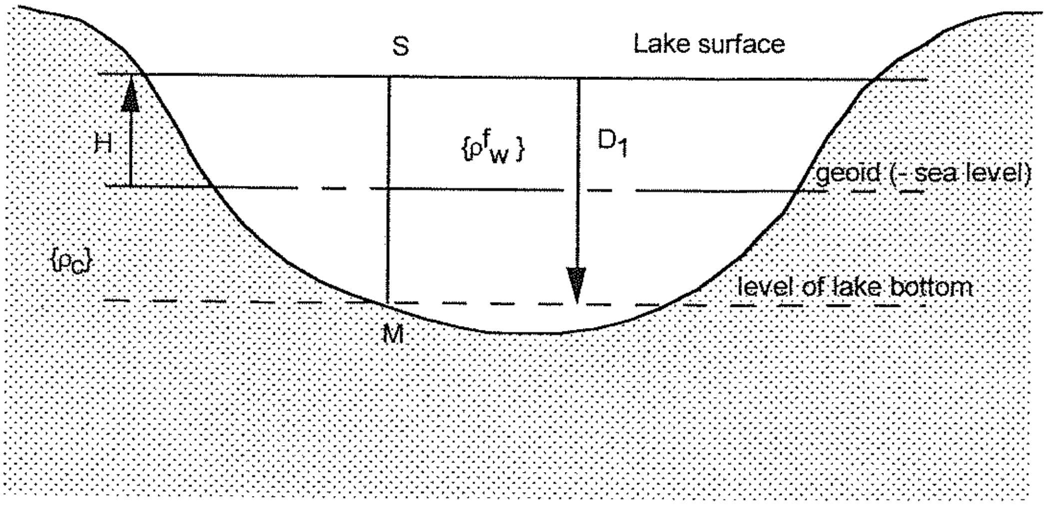
* *FA : The principle is to compare the gravity of the Earth at its surface with the normal gravity, which first requires in some cases to derive the surface value from the measured value. Then, and until now, FA is the difference between this Earth's gravity value reduced to the geoid and the normal gravity γ_o computed on the reference ellipsoid (classical concept). The more modern concept *, in which the gravity anomaly is the difference between the gravity at the surface point and the normal (ellipsoidal) gravity on the telluroid corresponding point may be adopted in the future depending on other major changes in the BGI data base and data management system.*

* *BO : The basic principle is to remove from the surface gravity the gravitational attraction of one (or several) infinite plate (s) with density depending on where the plate is with respect to the geoid. The conventional computation of BO assumes that parts below the geoid are to be filled with crustal material of density ρ_c and that the parts above the geoid have the density of the existing material (which is removed).*

---

* cf. "On the definition and numerical computation of free air gravity anomalies", by H.G. Wenzel. Bulletin d'Information, BGI, n° 64, pp. 23-40, June 1989.

<u>For example,</u> if a measurement g_M is taken at the bottom of a lake, with the bottom being below sea level, we have :



$$g_S = g_M + 2\,k\,\rho^f_w\,D_1 - \Gamma D_1$$

$$\Rightarrow FA = g_S + \Gamma H - \gamma_O$$

Removing the (actual or virtual) topographic masses as said above, we find :

$$\begin{aligned}\delta g_s &= g_s - k\,\rho^f_w\,D_1 + k\,\rho_c\left(D_1 - H\right)\\ &= g_s - k\,\rho^f_w\left[H + (D_1 - H)\right] + k\,\rho_c\,(D_1 - H)\\ &= g_s - k\,\rho^f_w\,H + k\,(\rho_c - \rho^f_w)(D_1 - H)\end{aligned}$$

$$\Rightarrow \quad BO = \delta g_s + \Gamma\,H - \gamma_o$$

The table below covers most frequent cases. It is an update of the list of formulas published before.

It may be noted that, although some formulas look different, they give the same results. For instance BO (C) and BO (D) are identical since :

$$\begin{aligned}-k\,\rho_l H + k(\rho_c - \rho_l)(D_1 - H) &\equiv -k\,\rho_l(H - D_1 + D_1) - k(\rho_c - \rho_l)(H - D_1)\\ &\equiv -k\,\rho_l\,D_1 - k\,\rho_c(H - D_1)\end{aligned}$$

Similarly, BO (6), BO (7) and BO (8) are identical.

Elev. Type	Situation	Formulas
EOL land data format		
1	Land Observation-surface	$FA = g + \Gamma H - \gamma_0$
		$BO = FA - k\,\rho_c\,H$
2	Land Observation-subsurface	$FA = g + 2\,k\,\rho_c\,D_2 + \Gamma(H - D_2) - \gamma_0$
		$BO = FA - k\,\rho_c\,H$
3	Lake surface above sea level	$FA = g + \Gamma H - \gamma_0$
	with bottom above sea level	$BO = FA - k\,\rho_w^f\,D_1 - k\,\rho_c\,(H - D_1)$
4	Lake bottom, above sea level	$FA = g + 2\,k\,\rho_w^f\,D_1 + \Gamma(H - D_1) - \gamma_0$
		$BO = FA - k\,\rho_w^f\,D_1 - k\,\rho_c\,(H - D_1)$
5	Lake bottom, below sea level	$FA = g + 2\,k\,\rho_w^f\,D_1 + \Gamma(H - D_1) - \gamma_0$
		$BO = FA - k\,\rho_w^f\,H + k\,(\rho_c - \rho_w^f)\,(D_1 - H)$
6	Lake surface above sea level	$FA = g + \Gamma H - \gamma_0$
	with bottom below sea level	$BO = FA - k\,\rho_w^f\,H + k\,(\rho_c - \rho_w^f)\,(D_1 - H)$
7	Lake surface, below sea level (here $H < o$)	$FA = g + \Gamma H - \gamma_0$
		$BO = FA - k\,\rho_c H + k\,(\rho_c - \rho_w^f)\,D_1$
8	Lake bottom, with surface below sea level ($H < o$)	$FA = g + (2\,k\,\rho_w^f - \Gamma)\,D_1 + \Gamma H - \gamma_0$
		$BO = FA - k\,\rho_c H + k\,(\rho_c - \rho_w^f)\,D_1$
9	Ice cap surface, with bottom below sea level	$FA = g + \Gamma H - \gamma_0$
		$BO = FA - k\,\rho_i\,H + k\,(\rho_c - \rho_i)\,(D_1 - H)$
10	Ice cap surface, with bottom above sea level	$FA = g + \Gamma H - \gamma_0$
		$BO = FA - k\,\rho_i\,D_1 - k\,\rho_c\,(H - D_1)$
EOS Sea Data Format		
1	Ocean Surface	$FA = g - \gamma_0$
		$BO = FA + k\,(\rho_c - \rho_w^s)\,D_1$
2	Ocean submerged	$FA = g + (2\,k\,\rho_w^s - \Gamma)\,D_2 - \gamma_0$
		$BO = FA + k\,(\rho_c - \rho_w^s)\,D_1$
3	Ocean bottom	$FA = g + (2\,k\,\rho_w^s - \Gamma)\,D_1 - \gamma_0$
		$BO = FA + k\,(\rho_c - \rho_w^s)\,D_1$

All requests for data must be sent to :

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In case of a request made by telephone, it should be followed by a confirmation letter, or fax. Except in particular case (massive data retrieval, holidays...) requests are satisfied within one month following the reception of the written confirmation, or information are given concerning the problems encountered.

If not specified, the data will be written as tarfiles on DAT cartridge (4 mm). for large amounts of data, or on diskette in the case of small files. The exact physical format will be indicated in each case. Also a FTP anonymous service is available on our computer center.

3. USUAL SERVICES BGI CAN PROVIDE

The list below is not restrictive and other services (massive retrieval, special evaluation and products...) may be provided upon request.

The costs of the services listed below are a revision of the charging policy established in 1981 (and revised in 1989) in view of the categories of users : (1) contributors of measurements and scientists, (2) other individuals and private companies.

The prices given below are in French Francs. They have been effective on January 1, 1992 and may be revised periodically.

3.1. Charging Policy for Data Contributors and Scientists

For these users and until further notice, - and within the limitation of our in house budget, we shall only charge the incremental cost of the services provided. In all other cases, a different charging policy might be applied.

However, and at the discretion of the Director of B.G.I., some of the services listed below may be provided free of charge upon request, to major data contributors, individuals working in universities, especially students ...

3.1.1. Digital Data Retrieval

. on CD-Roms : see 2.5.1.
. on one of the following media :
** printout .... 2 F/100 lines*
** diskette....................... 25 F per diskette (minimum charge : 50 F-*
** magnetic tape .... 2 F per 100 records*
+ 100 F per DAT cartridge
(if the tape is not to be returned)

. minimum charge : 100 F

. maximum number of points : 100 000 ; massive data retrieval (in one or several batches) will be processed and charged on a case by case basis.

3.1.2. Data Coverage Plots : in Black and White, with Detailed Indices

. 20°x20° blocks, as shown on the next pages (maps 1 and 2) : 400 F each set.

. For any specified area (rectangular configurations delimited by meridians and parallels) : 1 F per degree square : 100 F minimum charge (at any scale, within a maximum plot size of : 90 cm x 180 cm).

. For area inside polygon : same prices as above, counting the area of the minimum rectangle comprising the polygon.

3.1.3. Data Screening

(Selection of one point per specified unit area, in decimal degrees of latitude and longitude, i.e. selection of first data point encountered in each mesh area).

. 5 F/100 points to be screened.

. 100 F minimum charge.

3.1.4. Gridding

(Interpolation at regular intervals Δ in longitude and Δ' in latitude - in decimal degrees) :

. 10 F/($\Delta\Delta'$) per degree square

. minimum charge : 150 F

. maximum area : 40° x 40°

3.1.5. Contour Maps of Bouguer or Free-Air Anomalies

At a specified contour interval Δ (1, 2, 5,... mgal), on a given projection :
10 F/Δ per degree square, plus the cost of gridding (see 3.4) after agreement on grid stepsizes. (at any scale, within a maximum map size for : 90 cm x 180 cm).

. 250 F minimum charge

. maximum area : 40° x 40°

3.1.6. Computation of Mean Gravity Anomalies

(Free-air, Bouguer, isostatic) over Δ xΔ' area : 10F/ΔΔ' per degree square.

. minimum charge : 150 F

. maximum area : 40°x40°

3.2. Charging Policy for Other Individuals or Private Companies

3.2.1. Digital Data Retrieval

. on CD-Roms : see 2.5.1.

. 1 F per measurement for non commercial use (guaranteed by signed agreement), 5 F per measurement in other cases (direct or indirect commercial use - e.g. in case of use for gridding and/or maps to be sold or distributed by the buyer in any project with commercial application). Minimum charge : 500 F

3.2.2. Data Coverage Plots, in Black and White, with Detailed Indices

. 2 F per degree square ; 100 F minimum charge. (maximum plot size = 90 cm x 180 cm)

. For area inside polygon : same price as above, counting the area of the smallest rectangle comprising ihe polygon.

3.2.3. Data Screening

. 1 F per screened point for non commercial use (guaranteed by signed agreement), 5 F per screened point in other cases (cf. 3.2.1.).

. 500 F minimum charge

3.2.4. Gridding

Same as 3.1.4.

3.2.5. Contour Maps of Bouguer or Free-Air Anomalies

Same as 3.1.5.

3.2.6. Computation of Mean Gravity Anomalies

Same as 3.1.6.

3.3. Gravity Maps

The pricing policy is the same for all categories of users

3.3.1. Catalogue of all Gravity Maps

Printout : 200 F
DAT cartridge (4 mm) 100 F

3.2.2. Maps

. Gravity anomaly maps (excluding those listed below) : 100 F each

. Special maps :

Mean Altitude Maps

FRANCE	(1: 600 000)	1948	6 sheets	65 FF the set
WESTERN EUROPE	(1:2 000 000)	1948	1 sheet	55 FF
NORTH AFRICA	(1:2 000 000)	1950	2 sheets	60 FF the set
MADAGASCAR	(1:1 000 000)	1955	3 sheets	55 FF the set
MADAGASCAR	(1:2 000 000)	1956	1 sheet	60 FF

Maps of Gravity Anomalies

NORTHERN FRANCE	Isostatic anomalies	(1:1 000 000)	1954	55 FF
SOUTHERN FRANCE	Isostatic anomalies Airy 50	(1:1 000 000)	1954	55 FF
EUROPE-NORTH AFRICA	Mean Free air anomalies	(1:1 000 000)	1973	90 FF

World Maps of Anomalies (with text)

PARIS-AMSTERDAM	Bouguer anomalies	(1:1 000 000)	1959-60	65 FF
BERLIN-VIENNA	Bouguer anomalies	(1:1 000 000)	1962-63	55 FF
BUDAPEST-OSLO	Bouguer anomalies	(1:1 000 000)	1964-65	65 FF
LAGHOUAT-RABAT	Bouguer anomalies	(1:1 000 000)	1970	65 FF
EUROPE-AFRICA	Bouguer Anomalies	(1:10 000 000)	1975	180 FF with text 120 FF without text
EUROPE-AFRICA	Bouguer anomalies-Airy 30	(1:10 000 000)	1962	65 FF

Charts of Recent Sea Gravity Tracks and Surveys (1:36 000 000)

CRUISES prior to	1970	65 FF
CRUISES	1970-1975	65 FF
CRUISES	1975-1977	65 FF

Miscellaneous

CATALOGUE OF ALL GRAVITY MAPS

listing	200 FF
tape	300 FF

THE UNIFICATION OF THE GRAVITY NETS OF AFRICA

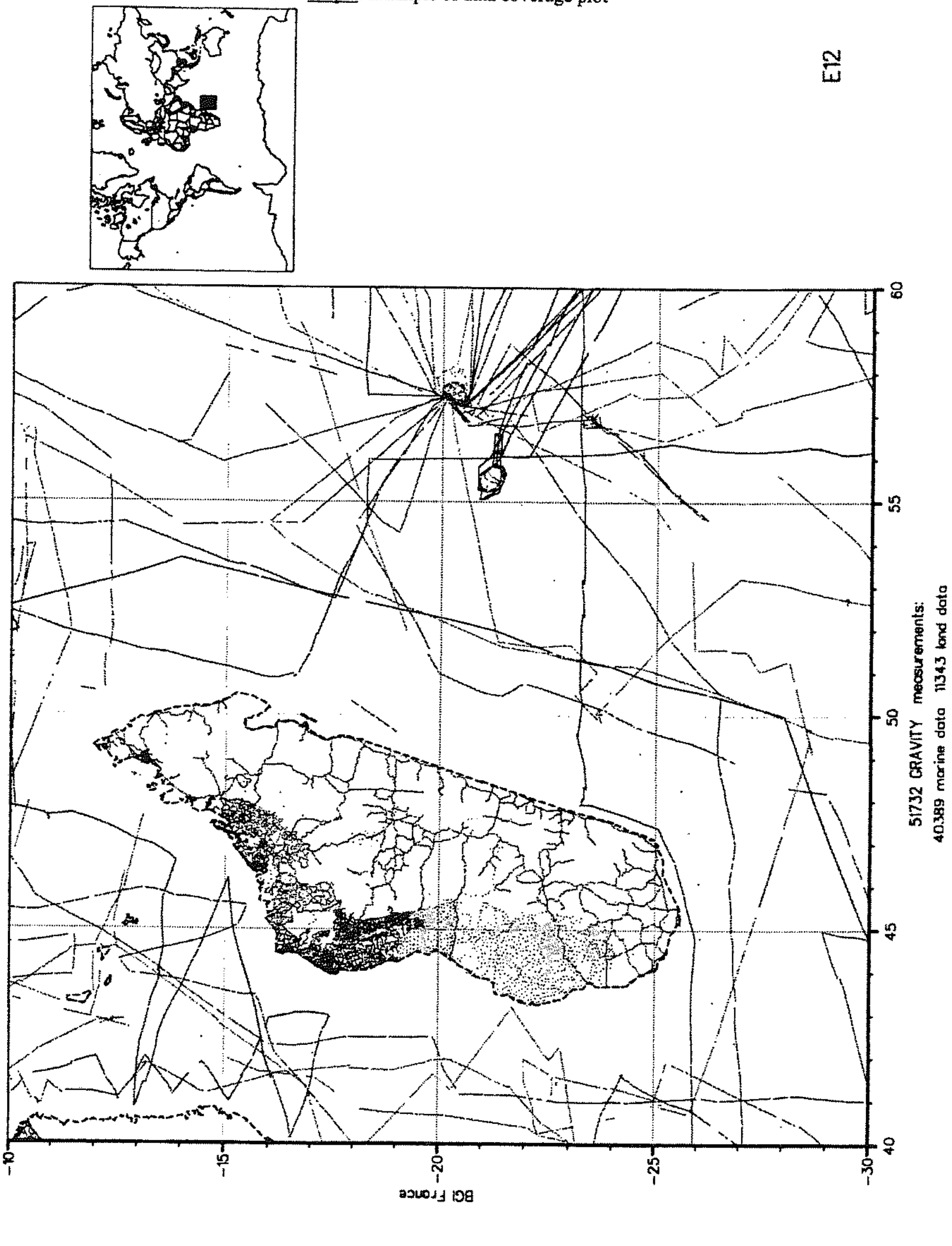
(Vol. 1 and 2)	1979	150 FF

. Black and white copy of maps : 150 F per copy

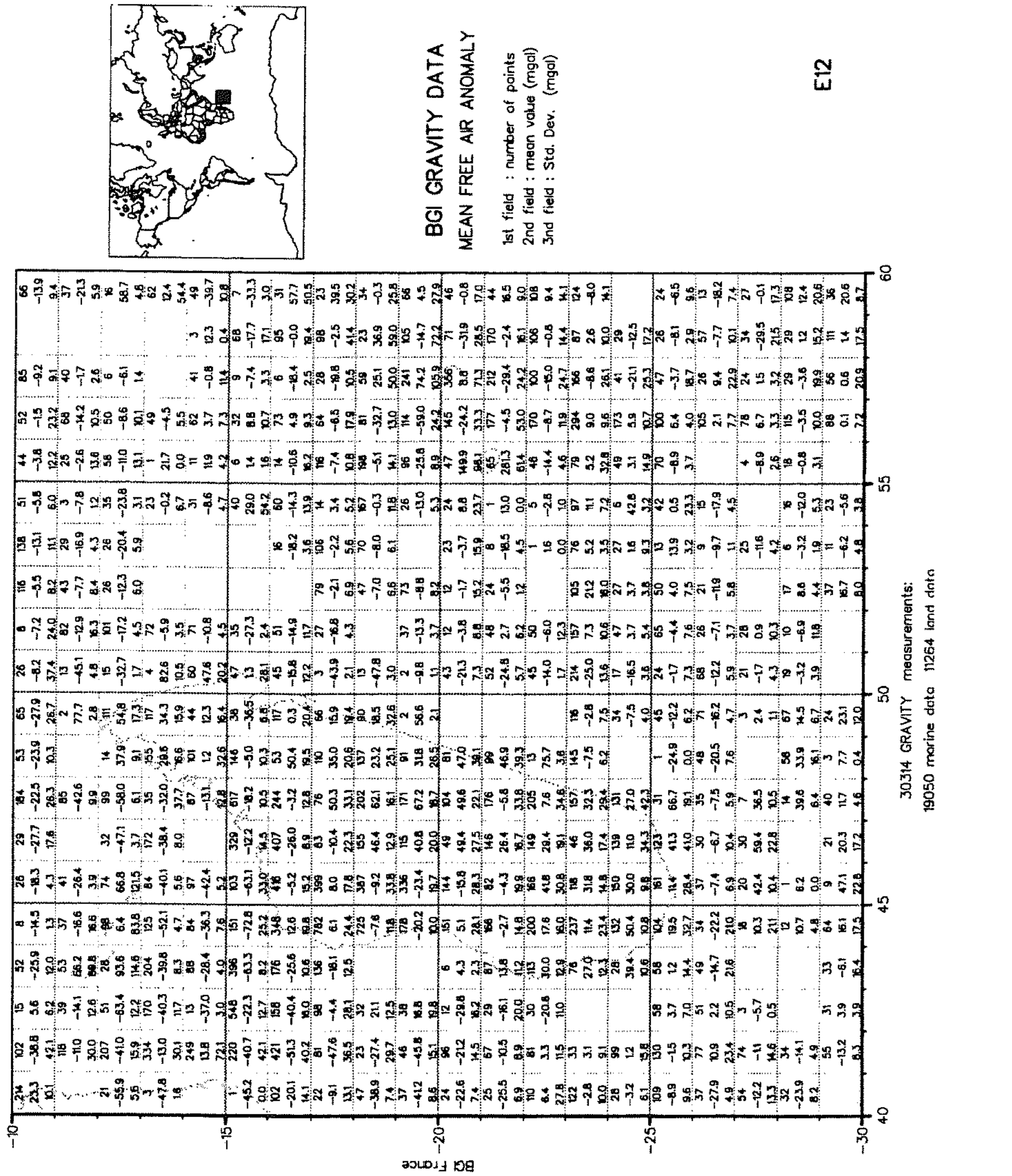
. Colour copy : price according to specifications of request.

Mailing charges will be added for air-mail parcels when "Air-Mail" is requested)

Map 1. Example of data coverage plot



Map 2. Example of detailed index (Data coverage corresponding to Map 1)



4. PROVIDING DATA TO B.G.I.

4.1. Essential Quantities and Information for Gravity Data Submission

1. Position of the site :

- *latitude, longitude (to the best possible accuracy),*
- *elevation or depth :*
  - *. for land data : elevation of the site (on the physical surface of the Earth)* *
  - *. for water stations : water depth.*

2. Measured (observed) gravity, corrected to eliminate the periodic gravitational effects of the Sun and Moon, and the instrument drift **

3. Reference (base) station (s) used. For each reference station (a site occupied in the survey where a previously determined gravity value is available and used to help establish datum and scale for the survey), give name, reference station number (if known), brief description of location of site, and the reference gravity value used for that station. Give the datum of the reference value ; example : IGSN 71.

4.2. Optional Information

The information listed below would be useful, if available. However, none of this information is mandatory.

. Instrumental accuracy :

- *identify gravimeter (s) used in the survey. Give manufacturer, model, and serial number, calibration factor (s) used, and method of determining the calibration factor (s).*
- *give estimate of the accuracy of measured (observed) gravity. Explain how accuracy value was determined.*

. Positioning accuracy :

- *identify method used to determine the position of each gravity measurement site.*
- *estimate accuracy of gravity station positions. Explain how estimate was obtained.*
- *identify the method used to determine the elevation of each gravity measurement site.*
- *estimate accuracy of elevation. Explain how estimate was obtained. Provide supplementary information, for elevation with respect to the Earth's surface or for water depth, when appropriate.*

. Miscellaneous information :

- *general description of the survey.*
  date of survey : organization and/or party conducting survey.
- *if appropriate : name of ship, identification of cruise.*
- *if possible, Eötvös correction for marine data.*

. Terrain correction

Please provide brief description of method used, specify : radius of area included in computation, rock density factor used and whether or not Bullard's term (curvature correction) has been applied.

---

* Give supplementary elevation data for measurements made on towers, on upper floor of buildings, inside of mines or tunnels, atop glacial ice. When applicable, specify whether gravity value applied to actual measurement site or it has been reduced to the Earth's physical surface (surface topography or water surface) Also give depth of actual measurement site below the water surface for underwater measurements.

** For marine gravity stations, gravity value should be corrected to eliminate effects of ship motion, or this effect should be provided and clearly explained.

. Isostatic gravity

Please specify type of isostatic anomaly computed.
Example : Airy-Heiskanen, T = 30 km.

. Description of geological setting of each site

4.3. Formats

Actually, any format is acceptable as soon as the essential quantities listed in 4.1. are present, and provided that the contributor gives satisfactory explanations in order to interpret his data properly.

The contributor may use the EOL and/or EOS formats as described above, or if he wishes so, the BGI Official Data Exchange Format established by BRGM in 1976 : "Progress Report for the Creation of a Worldwide Gravimetric Data Bank", published in BGI Bull. Info, n° 39, and recalled in Bulletin n° 50 (pages 112-113).

If magnetifc tapes are used, contributors are kindly asked to use 1600 bpi, unlabelled tapes (if possible), with no password, and formatted records of possibly fixed length and a fixed blocksize, too. Tapes are returned whenever specified, as soon as they are copied

PART II

CONTRIBUTING PAPERS

Note of the editor

Due to technical problems, the contribution of Prof. M. Everaerts, Ph. Lambot,T. Van Hoolst, M. van Ruymbeke and B. Ducarme, "First Order Gravity Network of Belgium", was not properly printed in the bulletin n° 89. A figure was missing and an other one was unreadable on some of the copies. I must apologise for the harm the authors have been caused. The paper is given once again in this issue.

H. Duquenne

First Order Gravity Network of Belgium

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Summary

Between 1998 and 2001 the Royal Observatory of Belgium and the National Geographic Institute of Belgium performed several gravity campaigns to establish a new Belgian Gravity Base Network (BLGBN98).
There are 41 base points. The scale is well constrained by 8 absolute gravity stations.
Nine gravimeters (LaCoste & Romberg and Scintrex) have been used on the field.
The data have been reduced in a common adjustment.
A scale factors has been determined for each instrument.
The RMS error on the unit weight reaches 19µgal.
The RMS error on the gravity points is ranging between 4µgal and 10µgal.
The results show the distorsions of a previous reference network realised in 1978.

1. Introduction

The goal was to establish a new fundamental gravity network (Figure 1) with a scale constrained by a maximum of absolute gravity measurements and a precision better than 10µgal, in order to replace a network observed in 1978 with only one absolute gravity point.
This network is a result of a close cooperation between the Royal Observatory of Belgium (ROB) and the National Geographic Institute (NGI) who organised several observation campaigns between 1998 and 2001 (Table 1). It benefited from the cooperation of several Belgian and foreign Institute who provided gravimeters (Table 2). We are especially indebted to the "*Institut für Physicalische Geodäsie, Technische Universität Darmstadt*" (IPG-TUD) who provided also an experienced field operator.
We greatly benefited also of the work realised since 1995 by the Royal Observatory of Belgium to establish a dense network of absolute gravity stations.
We used altogether 9 gravimeters on the field, 2 Scintrex and 7 model D or G LaCoste & Romberg. However only 4 instruments (D31, D32, S265 and G336) did effectively observe the complete network. Moreover S265 was sent back to the maker in 2000 and its scale factor was modified.
The network is constrained by 9 absolute gravity stations established by the ROB in Belgium and the neighbouring countries, using a FG5 absolute gravimeter with a nominal precision of 1µgal. The local gravity gradient has been measured carefully with the S265 instrument. The absolute stations cover the total range of gravity variations i.e. 260mgal.
For this survey all the points, except the absolute ones, were located outside buildings to keep them permanently accessible. In most of the cases church porches (Figure 2) were chosen for two reasons:
* those places have a high probability to be not altered in the near future;
* generally levelling benchmarks already exist in the vicinity.
The network includes 41 base-stations and their excentric points. The absolute gravity points were included when possible or introduced as excentric points directly connected to the closest station of the network. Altogether some 60 stations were occupied and more than 1,050 ties were measured.
From SE to NW the maximum gravity difference reaches 260mgal between Arlon and Meer.

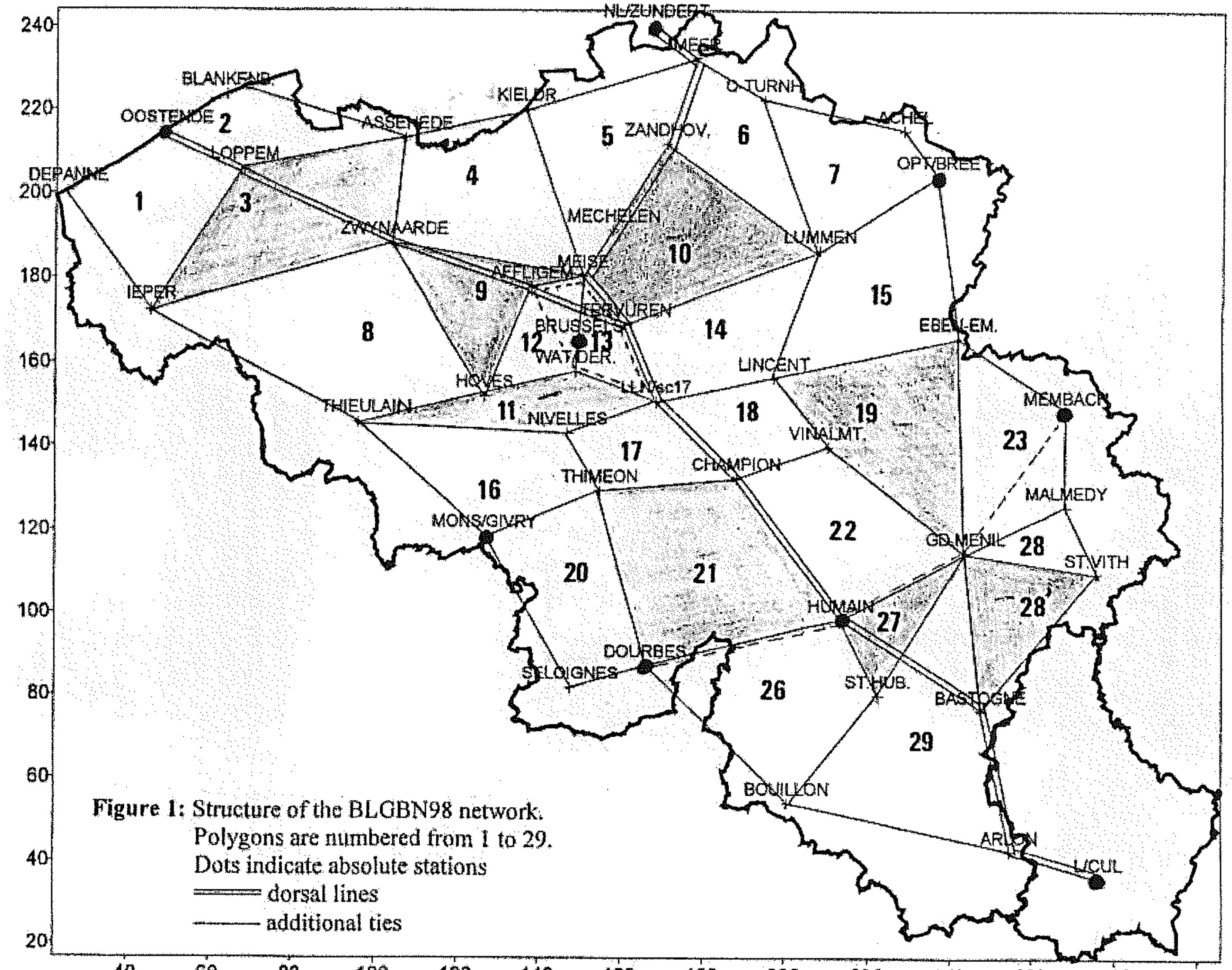


Figure 1: Structure of the BLGBN98 network.
Polygons are numbered from 1 to 29.
Dots indicate absolute stations
═══ dorsal lines
——— additional ties

TABLE 1
List of campaigns

ROB & IPG-TUD	NGI
	LCR D31 and D32
3 Days August 98 (S265,G336,G402,G487);	20 days in September and October 99
9 Days May 99 (S265,S342,D38,G258,G336,G402);	
3 Days June 99 (S265,S342,G402);	
4 Days September 99 (G336,G402);	
6 Days October 99 (S265,G336,G402);	
7 Days July 2001 (S265,G336,G487).	

total 162 "Gravimeter Days"

TABLE 2
List of Instruments:

Scintrex S265, LCR G402, LCR G336 Royal Observatory of Belgium (ROB)
LCR G487 Metrological Service of Belgium
LCR D31 Université catholique de Louvain et Université de Liége
LCR D32 National Geographical Institut (NGI)
Scintrex S342 Université de La Rochelle
LCR D38, G268 Institut für Physicalische Geodäsie, Technische Universität Darmstadt

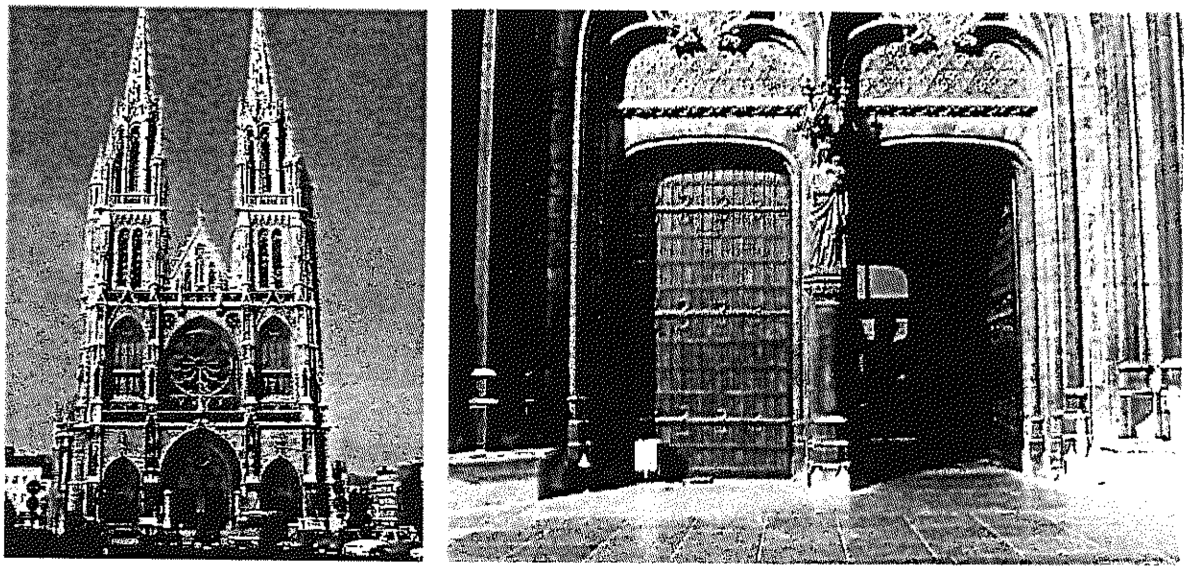
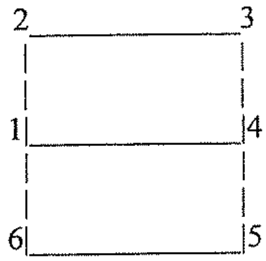


Figure 2: Gravity observations at Oostende station

2. Structure of the network (Figure 1)

To ensure the independence of the ties inside a loop the optimal solution should be to link each station with its direct neighbours in a sequence 1-2-1. Of course with 41 stations it is a very heavy task and we had not enough manpower to follow this schedule.

• The base network is subdivided in 27 polygons with 4 stations each. Two polygons are observed on the same day in a sequence 1-2-3-4-1-6-5-4 with two closures on the common side. The gravity differences are thus correlated inside of the loops.



A standard working day consists thus in measuring 8 points and requires less than 12 hours. The optimal solution 1-2-1-4-1-6-1, 3-2-3-4-3, 5-4-5-6-5 should require 17 measurements. In any case a partial decorrelation is obtained when observing adjacent polygons.

• To strengthen the structure we observed several dorsal lines starting from Brussels to the North West, the North and the South East in a sequence 1-2-3-4-3-2-1 (═══ on figure 1). Each of them was observed at least two times.

To limit the observational task we also decided to omit 8 polygons (shaded area) which connect stations that are already observed in other polygons. The effect of this decision was that a few stations on the border of the network were observed only once and we shall see that after adjustment they exhibit a larger RMS error.
The NGI observed the complete network in September-October 1999 with 2 gravimeters. The stations located to the East of the line Meer-Arlon were occupied in May 1999 by ROB and IPG-TUD with 6 instruments. In October 1999 ROB completed the network with only 3 gravimeters.

3. Measuring techniques

The measuring technique was slightly different for the Scintrex and the LaCoste & Romberg (LCR) gravimeters.

3.1 The Scintrex instruments

We use a 60s integration with continuous tilt adjustment and automatic rejection of bad data (Scintrex manual, 1992; Ducarme & Somerhausen, 1997).
The instrument is installed on the site 10 minutes before starting measurements for temperature adjustment. We perform at least 5 measurements. The tidal correction is applied during the offline data reduction (§4.1). The residual temperature effect is corrected online by the internal software.

3.2 The LCR instruments

We do not perform optical readings as the resolution is not sufficient for our purpose.
For all gravimeters we have access to an analogic signal in mV or Hz proportional to the difference between the real beam position and the reading line (van Ruymbeke, 1991; van

Ruymbeke & al., 1995). This signal is linear on a larger than 1mgal range. We do not try to zero exactly the gravimeter but we always use full divisions on the dial and correct for the residual signal. It allows to reach a one microgal resolution even with the G meters (Ducarme & al., 1976).

-After the levelling, we do a coarse micrometer adjustment on the zero within one dial unit (10µgal) for model G or one counter unit (10µgal) for model D.
-We perform a 100µGal displacement on each side of this preliminary position to determine the conversion factor mV or Hz to micrometer units.
-We do a minimum of 3 measurements to within 10µgal of the zero and measure the residual signal.
This complete procedure requires up to 15 minutes after unclamping.

Aims of this procedure
-This procedure allows a 10 minutes stabilisation of the instrument before starting the precise measurements;
-We determine for each station the conversion factor from mV or Hz to counter units;
-The successive readings are used to check the stability of the instrument and to detect anomalous measurements, reject them and make some additional readings if required.

To speed up the procedure it is possible to perform only a series of 4 measurements in a sequence: zero, +100, -100, zero. Then the reduction should be performed by least square adjustment (§4.1.2).

4. Data reduction

The reduction of the data is performed in different steps
- **First step:** for each loop and each instrument we compute, at each station, the mean value converted to physical units and corrected of the tidal effects.

- **Second step:** for each instrument we calculate the drift for the different possible closures of the loop to compute the gravity difference for each tie. We select semi-automatically the independent ties of each loop by choosing the best closure. Anomalous ties can be rejected at that level.

-**Third step:** All selected ties for one or several instruments are collected in a file with the format required by the network adjustment programs.

Table 3
Comparison of standard and simplified procedures
Gravimeter GD-032, Maker's calibration factor: 1.06073

a) standard procedure
Conversion factor mV to micrometer unit: 0.07120
Mean Corrected Micrometer value: 125685.18
Mean value converted in µgal: 153411.33
Mean time for tidal correction: 9h 06m UT
Tidal correction (µgal): +21.06
Tidefree mean value: 153390.27

EPOCH	Raw Micrometer	SIGNAL (mV)	Corrected. Micrometer	RESIDUE (micom. Unit)
1999 10 01 09 03	125685	0.	125685.00	-.18*
1999 10 01 09 04	125785	1409.	125684.68	-.50+
1999 10 01 09 05	125585	-1400.	125684.68	-.50+
1999 10 01 09 05	125685	0.	125685.00	-.18
1999 10 01 09 06	125695	139.	125685.10	-.07
1999 10 01 09 06	125675	-139.	125684.90	-.21
1999 10 01 09 07	125685	-10.	125685.71	.53
standard deviation	0.37 micrometer unit			

+ calibration displacement not included in the mean
* eliminated reading

b) linear regression
REGRESSION ON 7 POINTS: CM = A + B *(RM - 125686)
A= 0.01 B= 0.071204
±.15 ±.000192
STANDARD DEVIATION S= .38281991

Mean Corrected Micrometer value: 125685.01
Maker's calibration factor: 1.2206
Mean value converted in µgal: 153411.12
Mean time for tidal correction: 9h 05m UT
Tidal correction (µgal): +21.06
Tidefree mean value: 153390.06

EPOCH	Raw Micrometer	SIGNAL (mV)	Corrected Micrometer	RESIDUE (microm. Unit)
1999 10 01 09 03	125685	0.	125685.00	-.01
1999 10 01 09 04	125785	1409.	125684.67	-.34
1999 10 01 09 05	125585	-1400.	125684.69	-.32
1999 10 01 09 05	125685	0.	125685.00	-.01
1999 10 01 09 06	125695	139.	125685.10	.09
1999 10 01 09 06	125675	-139.	125684.90	-.11
1999 10 01 09 07	125685	-10.	125685.71	.70
standard deviation	0.35 micrometer unit			

Table 4
Data reduction using the standard procedure
Gravimeter GR-487, Maker's calibration factor: 1.0255

Conversion factor mV to micrometer unit: 0.0891
Mean Corrected Micrometer value: 4612838.10
Mean value converted in µgal: 4730465.47
Tidal correction (µgal): -34.48
Tidefree mean value: 4730499.94

EPOCH	Raw Microm.	SIGNAL (mV)	Corrected Micrometer	RESIDUE (dial units)
2001 07 03 07 02	4612820	-55.	4612824.90	-13.20*
2001 07 03 07 04	4612920	1005.	4612830.47	-7.63+
2001 07 03 07 06	4612720	-1240.	4612830.47	-7.63+
2001 07 03 07 08	4612830	-78.	4612836.95	-1.15
2001 07 03 07 10	4612840	22.	4612838.04	-.06
2001 07 03 07 13	4612830	-100.	4612838.91	.81
2001 07 03 07 15	4612840	17.	4612838.49	.39
standard deviation	0.84 dial unit			

+ calibration displacement
* eliminated reading

4.1 Mean corrected value

The tidal correction is computed for the mean epoch, as the tidal changes are quite linear on a few tens of minutes. What is more important is the use of regional tidal parameters as they can differ from a constant tidal factor by more than 10%. For a loop of 6 hours it means differences of more than 10µgal. We also use the same coordinates for all the stations belonging to the same loop.
For Scintrex instruments the mean value of the different readings is directly computed. The discrepancy between each reading and this mean is evaluated to detect outliers.
The reduction of the LaCoste&Romberg gravimeters is possible according to different schemes.
4.1.1 standard procedure (Tables 3a and 4)
For each displacement we compute the exact value by subtracting the residual signal multiplied by the conversion factor derived from the large displacements. These large excursions are not taken into account for the computation of the mean value as well as the preliminary reading which is only a coarse adjustment.
4.1.2 least square adjustment (Table 3b)
It is also possible to perform directly a linear regression between the residual signal and the different values of the micrometer. The slope gives the sensitivity and the independent term the crossing of the zero i.e. the true value of the micrometer.
Comparison of Tables 3a and 3b shows that the two procedures give the same result for the individual readings. The difference for the mean value is due to the different choice of the included readings. However they generally agree within the associated standard deviation. A larger disagreement should be interpreted as a sign of instability.

The simplified procedure (4 readings only) gives excellent results if the gravimeter is not drifting after unclamping as in the previous example. However for some instruments the first value can be quite different from the following ones (Table 4) and should indeed be suppressed and then 3 readings will not insure the required precision. It's why we normally always use at least 3 readings after the 2 calibration displacements and normally discard the preliminary reading.

Table 5
Computation of the different closures
Connection between station Givry (7041) and absolute point in Mons (7000)
Gravimeter Scintrex 265

STATION	EPOCH	RAW VALUE (μgal)	DRIFT CORR. VALUE (μgal)	gravity diff. (μgal)
closure 1 on station 7000	:drift/day = -128.8			
7000	2001 07 20 08 00	5325738.75	5325738.75	
				-7902.15
7041	2001 07 20 08 59	5317831.29	5317836.60	
				7902.15
7000	2001 07 20 10 02	5325727.85	5325738.75	
+				
*closure 2 on station 7000	:drift/day = -158.2			
7000	2001 07 20 08 00	5325738.75	5325738.75	
				-7900.93
7041	2001 0720 08 59	5317831.29	5317837.82	
				7903.43
7000	2001 07 20 10 02	5325727.85	5325741.25	
				-7903.38
7041	2001 07 20 10 53	5317818.85	5317837.86	
				7900.89
7000	2001 07 20 11 47	5325713.76	5325738.75	
+				
closure 3 on station 7041	:drift/day = -157.6			
7041	2001 07 20 08 59	5317831.29	5317831.29	
				7903.40
7000	2001 07 20 10 02	5325727.85	5325734.69	
				-7903.40
7041	2001 07 20 10 53	5317818.85	5317831.29	
+				
closure 4 on station 7000	:drift/day = -192.3			
7000	2001 07 20 10 02	5325727.85	5325727.85	
				-7902.17
7041	2001 07 20 10 53	5317818.85	5317825.68	
				7902.17
7000	2001 07 20 11 47	5325713.76	5325727.85	
+				

* selected closure

4.2 Drift computation and selection of the ties

For each loop the program identifies all possible closures for computation of a linear instrumental drift (Table 5).
For each loop the best independent closures are manually selected. In the case of reiterated closures on a same point it is possible to select successive closures or the global one as in the example of Table 5. The choice will depend on the linearity of the drift on the complete observation span. If the global solution is rejected, care should be taken at that level not to keep duplicated ties in two independent closures.

4.3 Preparation of the input files for least square adjustment

We are using two adjustment softwares, ADJNODE (Ph.Lambot) and ADJG (Jiang, 1988), which require different input format. We had to write the program FORMADJ to rewrite the ties in the correct format. This program is also able to mix and sort ties of different instruments as well as to make statistics on the ties between each pair of stations.

5. Network adjustment

The adjustment of the network is first performed for each instrument independently to check the internal coherency, before computing a global compensation including the determination of individual scale factors for the gravimeters, the scale of the network being controlled through several absolute points with large gravity differences.

5.1 Individual adjustments: For each gravimeter we perform an adjustment of the gravity values with reference to a fixed point to detect the gross errors in the observations. For this purpose we use the software "ADJNODE". Besides gravity differences with respect to the fixed point and the associated RMS errors this least square adjustment computes the residual for each tie. We eliminate the ties with residues larger than 3 times the RMS error on the unit weight. In table 6 we give the RMS error associated with each gravimeter. As the program is normally used for the adjustment of the nodes in a network observed in a way similar to a levelling network, the ties are supposed to be independent. As already noticed our ties are correlated inside of a loop and it reduces the estimated errors in the least square adjustment. This correlation will largely disappear in the global adjustment due to the mixture of several instrument. The estimated errors on each instrument will increase accordingly.

5.2 Global adjustment: We perform a final adjustment with all the instruments constrained by the absolute gravity values. For that purpose we adapted the software "ADJG" (Jiang & al., 1988; Ziang, 1999). The weight of the absolute gravity values can be adjusted according to their estimated accuracy. For each gravimeter we can compute a polynomial adjustment of its scale factor. For LCR D meters we can, of course, compute only a constant scale factor as the readings depend from the reset adjustments. Moreover the ADJG software allows also the determination of cyclic micrometer errors but we did not use this option.
The standard output provides for each station, including the absolute ones, the adjusted gravity value with its RMS error. The discrepancy between the adjusted and observed value of the gravity points should not exceed the associated RMS error.
For each gravimeter we get also the RMS error on the residuals for each tie giving an estimation of the precision of the instrument. The sum of the residuals indicates if any bias is present for a given instrument.

Table 6
INDIVIDUAL ERROR ESTIMATION

GRAV.	RMS err. (μgal)	GRAV	RMS err. (μgal)	GRAV	RMS err. (μgal)
GD- 31 :	16.4	GD- 32	: 10.1	GD- 38	: 14.4
GR-258 :	20.7	SC-265*	: 9.9	GR-336	: 11.2
SC-342 :	13.5	GR-402	: 13.7	GR-487	: 17.9

* before revision in 2000

6. Final Adjustment

Our final adjustments incorporate more than 1050 ties observed with the 9 gravimeters.
It was constrained by up to 9 absolute gravity values.
Each tie has a unit weight and the absolute gravity values an adjustable weight P, normally equal to 4. This choice is justified a posteriori by the fact that the RMS error on the absolute points is close to 5μgal compared to 20μgal for a single tie.
For each gravimeter we computed a single scale factor. Polynomial adjustment of the scale did not provide results statistically better.
To reduce the internal errors we rejected ties with a residue higher than three times the observed standard deviation on the unit weight. We suppressed so about 1% of the ties. The standard deviation was reduced from 25μgal to 20μgal, without any significant change in the solution.
There are two degrees of freedom in the solution i.e. the number of absolute gravity points included and their weights. We thus have to select the best solution taking into account the following criteria.

- The sum of the residues on the links after adjustment should be as close as possible to zero. If not, there exists a strong distorsion in the solution.
- The RMS errors on the computed gravity values should be minimum.
- At the absolute gravity points, the difference between the computed value G and the nominal value g should be less than the associated RMS error M.

7. Selected solution

The first criterion is always satisfied.
However there is a conflict between the absolute station of Humain (HUM) in the South-East of Belgium and the neighbouring absolute stations of Dourbes (DOU), Luxembourg (LUA) and Membach (MEM). Moreover the tie between Arlon and Luxembourg is still weak.

- The solution including the 9 absolute stations with an equal weight (Table 7) is violating the criterion concerning the absolute stations with a difference of -20μgal between the computed **G** and the a priori **g** values at Humain and large residues with opposite sign on the three conflicting stations. However this solution gives the lowest RMS errors on the stations in eastern Belgium.

TABLE 7: **CORRECTIONS TO THE ABSOLUTE POINTS**

in units of mgal

No	POINT	Sn	g observed	G adjusted	G-g	RMS error
1	BRU	105.00	981128.877	981128.877	.000	.005
2	LUA	151.10	980960.407	980960.415	.008	.007
3	NLZ	232.20	981196.849	981196.853	.004	.007
4	BRE	396.01	981149.022	981149.016	-.007	.006
5	MEM	483.71	981046.730	981046.737	.007	.006
6	DOU	567.00	981018.151	981018.159	.008	.006
7	**HUM**	**690.01**	**981002.122**	**981002.102**	**-.020**	**.005**
8	MNS	700.00	981082.876	981082.874	-.002	.006
9	OST	840.01	981173.303	981173.304	.001	.007

TABLE 8: **CORRECTIONS TO THE ABSOLUTE POINTS**

Station Humain (HUM) eliminated

in units of mgal

No	POINT	Sn	g observed	G adjusted	G-g	RMS error
1	BRU	105.00	981128.877	981128.876	-.001	.005
2	LUA	151.10	980960.407	980960.409	.001	.008
3	NLZ	232.20	981196.849	981196.855	.006	.007
4	BRE	396.01	981149.022	981149.015	-.007	.006
5	MEM	483.71	981046.730	981046.732	.002	.006
6	DOU	567.00	981018.151	981018.152	.001	.006
7	MNS	700.00	981082.876	981082.872	-.004	.007
8	OST	840.01	981173.303	981173.305	.002	.007

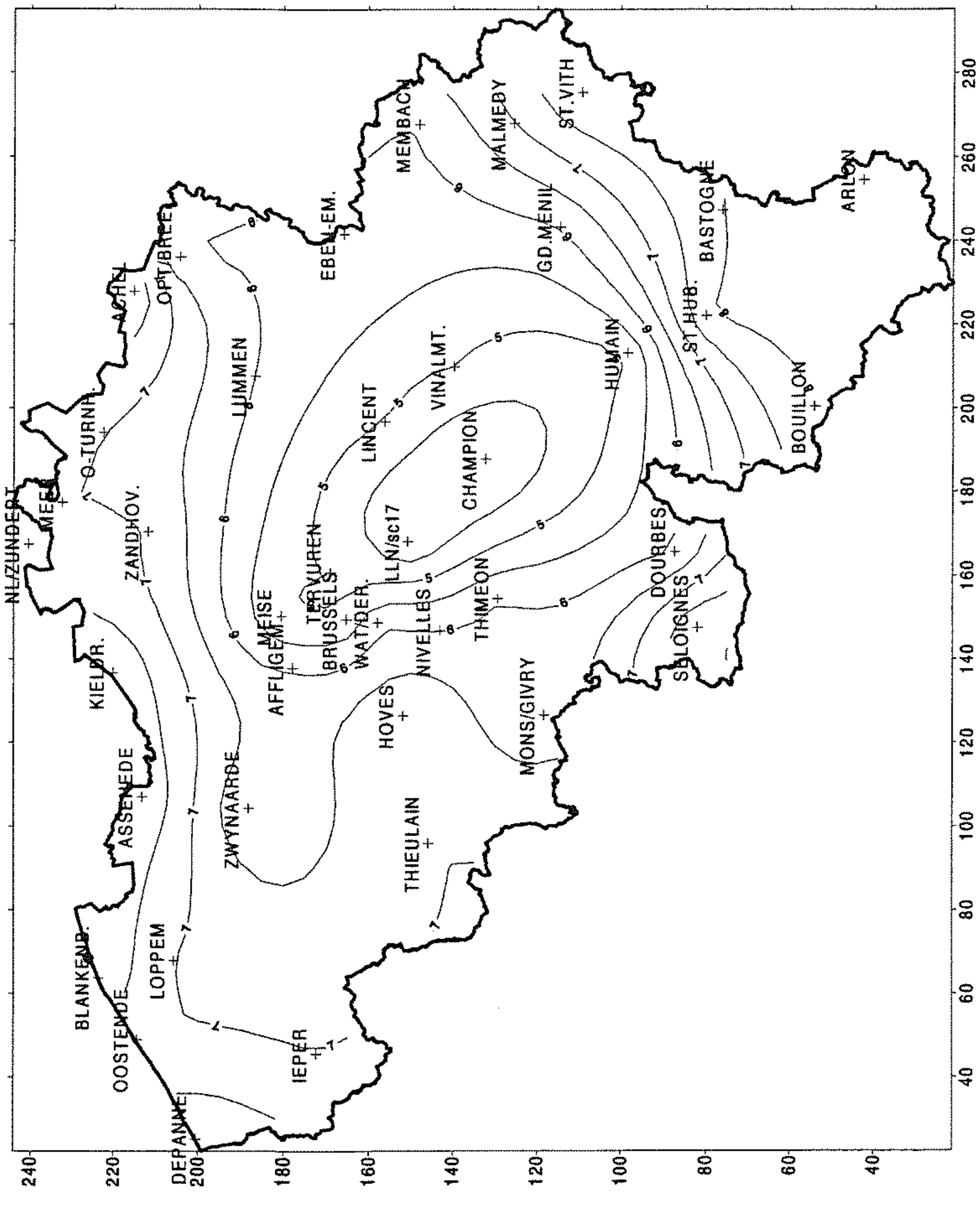


Figure 3: Repartition of the RMS error on the gravity values expressed in µgal

• The solution excluding Humain (Table 8) fulfils the criterion on the absolute stations but with slightly higher RMS errors in the Eastern part of the network. At Humain the difference between the adjusted gravity **G** and the observed gravity **g** reaches 29µgal, clearly indicating a systematic error.
From SE to NW the difference between the two solutions reaches 11µgal in Arlon, decreases to 5µgal in Champion, 1µgal in Brussels and changes its sign to -2µgal in Meer. It means that most of the network is constrained to better than 5µgal.
The absolute value in Humain is certainly questionable and should be rejected from the adjustment. This conviction is corroborated by the fact that this station had also to be eliminated by ROB from the project "Soulèvement de l'Ardenne" for inconsistent reiteration results. We selected thus the second solution with only 8 absolute gravity values.

8. Repartition of errors on the gravity values

The errors on the gravity values are spatially correlated and ranging between 4µgal and 10µgal. On Figure 3 it is clearly seen that the repartition is influenced by the dorsal lines and the absolute points. It should be noted also that the points to the SW of the main dorsal line Arlon-Meer have been occupied by only 6 gravimeters.
Outside the province of Luxembourg the larger errors on the edges of the network correspond to stations, which have been occupied only once according to our schedule.
In the province of Luxembourg we notice a broad zone with 8µgal errors although Arlon and Bastogne are on the SE dorsal line. To strengthen the solution in the South-Eastern part of Belgium we should improve the tie with Luxembourg absolute point and probably install an absolute point in Arlon where we reach the lowest gravity value of the net.

9. Normalisation and internal errors for the gravimeters

• The normalisation factors computed for each gravimeter are given in Table 9. It should be noted that these factors are insensitive to the choice of the absolute points and remained constant in all the solutions.
The Scintrex gravimeter belonging to ROB has two different factors. Prior to a revision (SC265) it is affected by a calibration error of 0.1%. The new factor given by the manufacturer after revision (SC266) seems correct.
Several gravimeters require a normalisation factor:
D38 (.03%), G336 (.05%), SC342 and G402 (.08%)
Other instruments do not require adjustment:
D31, D32, G258, G487

• The RMS errors on the unit weight are very different (Table 9), ranging from 12.8µgal to 16.4µgal for the Scintrex instruments and 15.7µgal to 26.4µgal for the LaCoste ones.
Comparing with table 6 it should be noted that the errors, as expected, are larger in the global adjustment but that the hierarchy of the instrument is confirmed.
The D32 is exceptional. Not only it has the lowest internal error among the LaCoste gravimeters used but also no tie of this instrument had to be rejected.

10. Comparison with the previous network

In 1978 a base gravimetric network of 27 stations had been measured using 6 LaCoste & Romberg G and D gravity meters (Poitevin & Ducarme, 1980). It was referred to the absolute station measured at ROB in 1976 by the "Istituto de Metrologia G. Colonetti". No external

TABLE 9

THE NORMALISATION FACTORS and INTERNAL ERRORS

GRAV.	NORM. FACT.	N TIES	RMS error (µgal)
GD- 31 :	1.0002721 ±.0000780	103	21.8
GD- 32 :	1.0001578 ±.0000969	108	15.7
GD- 38 :	0.9997272 ±.0000849	63	22.0
GR-258 :	1.0001481 ±.0000870	47	26.4
SC-265 :	0.9990790 ±.0000677	213	12.8
SC-266*:	1.0005379 ±.0001969	43	(8.9)
GR-336 :	1.0004703 ±.0000727	179	18.3
SC-342 :	1.0008077 ±.0000763	97	16.4
GR-402 :	1.0008177 ±.0000704	153	19.9
GR-487 :	1.0001064 ±.0001012	55	25.2
GLOBAL		**1061**	**18.7**

*after revision in 2000 the SC265 was renamed SC266 for the partial survey performed in 2001

constraint was available for the scale determination and all instruments were scaled on the LCR008 which was the best instrument (Poitevin, 1980).

Only a few stations are common to both networks. On figure 4 we give the difference expressed in microgal between this old network and the new one.

Besides an offset of -12µgal due to the revision of the Brussels absolute value we clearly see an overall tilt from NW to SE of more than 100µgal. As this direction corresponds to the main gravity gradient it could be explained by a systematic scale error of 0.04%, which is not unlikely for a LaCoste & Romberg instrument.

11. Conclusions

- Around 1050 ties performed with 9 different instruments link the 41 base stations and their excentric points.
- The RMS error on the unit weight is 19µgal.
- The RMS errors on the gravity values are comprised between 4µgal and 10µgal.
- The solution is perfectly stable in most of the country except in the province of Luxembourg in the SE, where the maximum difference between two extreme solutions reaches 10µgal. It is due to an abnormal value in one of the nine absolute stations, conflicting with the surrounding ones. This anomalous station has clearly to be rejected.

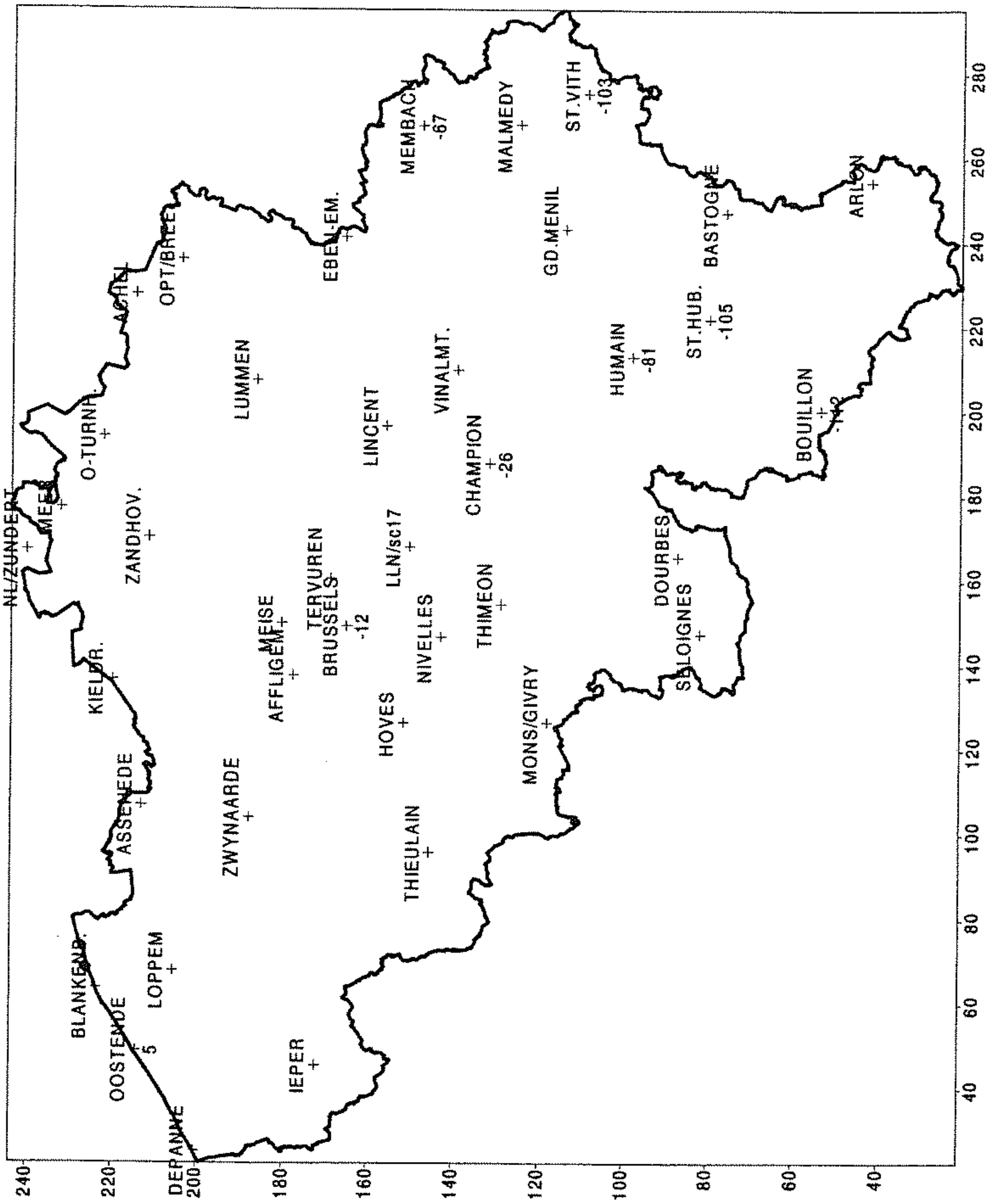


Figure 4: Difference in μgal between BLGBN98 and the previous 1978 network.

• The new gravity base network of Belgium is well constrained by the 8 remaining absolute gravity stations.
• We have now corrected the global distorsion of the previous network are thus able to perfectly unify all the local networks observed in Belgium since more than 10 years.

We recommend to improve the stability of the results in the SE corner of the network. For that purpose we are planning :
- To install in this area, i.e. in Arlon, an additional absolute point in a more stable station.
- To improve the relative gravity ties with the absolute station in Luxembourg city;

Acknowledgement:

We are very grateful to **O. Francis** and **M. Van Camp** who provided the absolute gravity values, to **G.Briffaut**, **A.El Wahabi**, **G.Laufer**, **R.Laurent** and **K.Monfils**, who were associated as field operators and to **Z. Jiang** who gave the adjustment software ADJG. **L. De Gols** encoded the LCR reading forms. **K. Gersteneker** lent us the gravimeters LCR D38 and G268, **A.Monjoie**, **C.Schroeder** and **J..F. Thimus** the gravimeter LCR D31 and **N. Florch** the Scintrex S342.
The Royal Observatory of Belgium and the National Geographic Institute gave a full support to the field operations.
The Belgian National Fund for Scientific Research provided his support through the grant FNRS/NFWO2.5.188.99 (S2/5-DD-E188).

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A NEW DERIVATION OF THE LEAST SQUARES COLLOCATION FORMULA

by J.P. BARRIOT, BGI

1. Introduction

The word "collocation" is a widely misused and misunderstood word, with a lot of meanings. Here this word is used in the sense of a linear "prediction" of a given quantity from another quantities, for example the matrix system $A\underset{\sim}{x} = \underset{\sim}{b}$ is a "prediction" or "collocation" of $\underset{\sim}{b}$ from $\underset{\sim}{x}$ through the linear relation *A*. Numerous introductions to this theory exist, but they have the default, - at least to my eyes – of not emphasing the links to the usual least-squares methods as taught in high-school. The following is such an approach.

2. Least-square methods and generalized least-square methods

In basic least-squares, one seeks to "solve" w.r.t. $\underset{\sim}{x}$ the system

$$A\underset{\sim}{x} = \underset{\sim}{b} \tag{1}$$

where the matrix A is non-square, usually when we have more data points $\underset{\sim}{b}$ than unknowns $\underset{\sim}{x}$. This is done my minimizing w.r.t. $\underset{\sim}{x}$ the objective function

$$Q = \left(A\underset{\sim}{x} - \underset{\sim}{b}\right)^T C_b^{-1} \left(A\underset{\sim}{x} - \underset{\sim}{b}\right) \tag{2}$$

where *A* is a (non)-square matrix linking the unknowns $\underset{\sim}{x}$ (to be solved) to the measurements $\underset{\sim}{b}$. The measurements $\underset{\sim}{b}$ are affected by errors, characterized by a given covariance matrix C_b (the so-called variance-covariance matrix of the errors on the unknowns). If the errors are uncorrelated, C_b is a diagonal matrix, and if all the error variances are equal, then $C_b = \sigma_b^2 I$ where *I* is the identity matrix.

The least-squares solution of (1), found in any basic textbooks (see also annexes), is

$$\underset{\sim}{x}^* = \left(A^T C_b^{-1} A\right)^{-1} A^T C_b^{-1} \underset{\sim}{b} \tag{3}$$

with the associated covariance matrix of $\underset{\sim}{x}^*$ being

$$\operatorname{cov}\left(\underset{\sim}{x}^*\right) = \left(A^T C_b^{-1} A\right)^{-1} \tag{4}$$

If $C_b = \sigma_b^2 I$, i.e. if the errors on $\underset{\sim}{b}$ are uncorrelated and equal, (3) and (4) reduces to

$$x^* = \left(A^T A\right)^{-1} A^T b \tag{5}$$

and $\operatorname{cov}\left(x^*\right) = \sigma_b^2 \left(A^T A\right)^{-1}$ (6)

One can remark that in this case the estimate x^* is independent of σ_b^2.

Formulas (3) to (6) implicitly assumes that $\left(A^T C_b^{-1} A\right)^{-1}$ and/or $\left(A^T A\right)^{-1}$ exist.

In many real cases, Eq. (3) is unable to provide correct (i.e. physically acceptable) estimates for x. In this case one has to add "physically sound" information from the outside world to Eq. (1). "Physically sound" information is provided by considering new equations, which we can call "super-data", on the form:

$$I\, x = 0 \tag{7}$$

i.e. we are looking to an x "close" to zero, where I is the identity matrix. 0 is a null-vector associated with a "noise" C_o, in the form of a diagonal, or non-diagonal matrix. Instead of C_o, one notes very often C_x, but the meaning is the same.

Adding the new equations (7) to $Ax = b$ is equivalent of writing, in block format

$$\left[\frac{A}{I}\right] x = \left[\frac{b}{0}\right] \Leftrightarrow A'\, x = b' \tag{8}$$

with the associated equivalent "noise" matrix $C_{b'}$

$$C_{b'} = \left[\begin{array}{c|c} C_b & 0 \\ \hline 0 & C_x \end{array}\right] \tag{9}$$

The least-squares solution of the system (8-9) corresponds to the minimisation w.r.t. x of

$$Q' = \left(A'\, x - b'\right)^T C_{b'}^{-1} \left(A'\, x - b'\right) \tag{10}$$

i.e. to

$$x^* = \left(A'^T C_{b'}^{-1} A'\right)^{-1} A'^T C_{b'}^{-1} b', \tag{11}$$

with the associated covariance matrix

$$\operatorname{cov}\left(x^*\right) = \left(A'^T C_{b'}^{-1} A'\right)^{-1}. \tag{12}$$

If one substitutes the block-format expressions

$\left[\frac{A}{I}\right]$ for A', $\left[\frac{b}{0}\right]$ for b' and $\left[\begin{array}{c|c} C_b & 0 \\ \hline 0 & C_x \end{array}\right]$ for $C_{b'}$

on (11) and (12) and develop the corresponding expressions, one founds

$$x^* = \left(\left[A^T \mid I\right] \left[\begin{array}{c|c} C_b^{-1} & 0 \\ \hline 0 & C_x^{-1} \end{array}\right] \left[\frac{A}{I}\right] \right)^{-1} \left[A^T \mid I\right] \left[\begin{array}{c|c} C_b^{-1} & 0 \\ \hline 0 & C_x^{-1} \end{array}\right] \left[\frac{b}{0}\right] \tag{13}$$

and

$$\operatorname{cov}\left(x^*\right) = \left(\left[A^T \mid I\right] \left[\begin{array}{c|c} C_b^{-1} & 0 \\ \hline 0 & C_x^{-1} \end{array}\right] \left[\frac{A}{I}\right] \right)^{-1} \tag{14}$$

which simplifies to

$$\underset{\sim}{x}^* = \left(A^T C_b^{-1} A + C_x^{-1}\right)^{-1} A^T C_b^{-1} \underset{\sim}{b} \tag{15}$$

and $\operatorname{cov}\left(\underset{\sim}{x}^*\right) = \left(A^T C_b^{-1} A + C_x^{-1}\right)^{-1}$. (16)

If $C_b = \sigma_b^2\ I$ and $C_x = \sigma_x^2\ I$ (the identity matrices are not necessarily of the same dimension), (15) and (16) simplify to

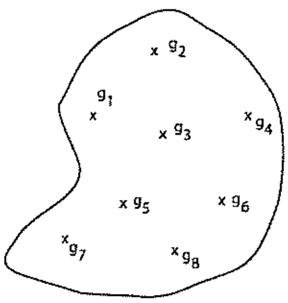
$$\underset{\sim}{x}^* = \left(A^T A + \frac{\sigma_x^2}{\sigma_b^2} I\right)^{-1} A^T \underset{\sim}{b} \tag{17}$$

and $\operatorname{cov}\left(\underset{\sim}{x}^*\right) = \left(A^T A + \frac{\sigma_x^2}{\sigma_b^2} I\right)^{-1}$, (18)

expressions which are often encountered in the practical world.

3. Collocation

One of the most simple example of collocation is the validation problem encountered in gravimetry, where one considers a set of gravity measurements over a network.



Network of gravimetric measurements.

One wants to make sure that there is no gross-error in the survey, i.e. to be sure that each measurement is coherent with respect to all others. Let be $\underset{\sim}{g} = \begin{bmatrix} g_1 \\ \vdots \\ g_N \end{bmatrix}$ the vector of the N measurements, affected by errors the covariance matrix of which is C_g (for example with $C_g = \sigma_g^2\ I$ if one supposes that all the measurements have "equal", uncorrelated errors).

Let $\underset{\sim}{\hat{g}}$ the N-vector of the "true" values (unknowns) of $\underset{\sim}{g}$. In a perfect world, with no measurements errors, we would have

$$\underset{\sim}{\hat{g}} = \underset{\sim}{g} \text{ i.e. } I\ \underset{\sim}{\hat{g}} = \underset{\sim}{g} \tag{19}$$

where I is the identity matrix.

Equation (19) is of the form of Eq. (1), and leads for the estimation of $\underset{\sim}{\hat{g}}$ from $\underset{\sim}{g}$ to

$$\underset{\sim}{\hat{g}}^* = \left(I\ C_g^{-1} I\right)^{-1} I\ C_g^{-1}\ \underset{\sim}{g} \text{ from (3),} \tag{20}$$

i.e. $\underset{\sim}{\hat{g}}^* = \underset{\sim}{g}$, (21) a clear non-sense for estimating errors on $\underline{g}$!

To obtain a no non-sense formula (i.e. not a tautology) one has to introduce the "super-data" of Eq (7), in the form

$$\underline{I}\,\hat{g} = \underline{0} \qquad (22)$$

where C_g and $C_{\hat{g}}$ are the corresponding covariance matrices, as in equations (8-16). $C_{\hat{g}}$ (not to be confounded with C_g, the matrix of errors in the measurements), and noted C_o or C_x in $(7-9)$ plays the role of a "link" between the values of g, which are "correlated" in the common sense, from place to place, i.e. if one has a large value of g in some place, one expects a large value of g in a neighbouring place, and vice versa.

With the introduction of the augmented system (22) (and therefore of the $C_{\hat{g}}$ covariance matrix), one has, on a manner equivalent to (15) and (16)

$$\underline{\hat{g}}^* = \left(I\,C_g^{-1} I + C_{\hat{g}}^{-1}\right)^{-1} I\,C_g^{-1}\,\underline{g} \qquad (23)$$

and $$\operatorname{cov}\left(\underline{\hat{g}}^*\right) = \left(I\,C_g^{-1} I + C_{\hat{g}}^{-1}\right)^{-1} \qquad (24)$$

i.e.

$$\underline{\hat{g}}^* = \left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} C_g^{-1}\,\underline{g} \qquad (25)$$

and $$\operatorname{cov}\left(\underline{\hat{g}}^*\right) = \left(C_g^{-1} I + C_{\hat{g}}^{-1}\right)^{-1} \qquad (26)$$

One can note, that in the case of no "measurement errors", i.e. $C_g = \sigma_g^2\, I, \quad \sigma_g^2 \to 0$

$$\underline{\hat{g}}^* = \left(\frac{1}{\sigma_g^2} I + C_{\hat{g}}^{-1}\right)^{-1} \frac{1}{\sigma_g^2} I\,\underline{g} \qquad (27)$$

$= \underline{g}$, as in formula (21)

The usual collocation formula is not found in textbooks as (25), i.e.

$$\underline{\hat{g}}^* = \left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} C_g^{-1}\,\underline{g} \qquad (28)$$

but as

$$\underline{\hat{g}}^* = C_{\hat{g}}\left(C_{\hat{g}} + C_g\right)^{-1}\underline{g} \qquad (29)$$

which is better, as the computation of the inverses of C_g and $C_{\hat{g}}$ is not needed.

In effect, (25) is equivalent to

$$\left(C_g^{-1} + C_{\hat{g}}^{-1}\right)\underline{\hat{g}}^* = C_g^{-1}\,\underline{g}$$

$$\Leftrightarrow C_g\left(C_g^{-1} + C_{\hat{g}}^{-1}\right)\underline{\hat{g}}^* = \underline{g}$$

$$\Leftrightarrow \underline{\hat{g}}^* + C_g C_{\hat{g}}^{-1}\,\underline{\hat{g}}^* = \underline{g}$$

$$\Leftrightarrow C_{\hat{g}} C_{\hat{g}}^{-1}\hat{g}^* + C_g C_{\hat{g}}^{-1}\hat{g}^* = \underline{g}$$

$$\Leftrightarrow \left(C_{\hat{g}} + C_g\right) C_{\hat{g}}^{-1}\hat{g}^* = \underline{g}$$

$$\Leftrightarrow C_{\hat{g}}^{-1}\,\underline{\hat{g}}^* = \left(C_{\hat{g}} + C_g\right)^{-1}\underline{g}$$

$$\Leftrightarrow \hat{\underset{\sim}{g}}^* = C_{\hat{g}}\left(C_{\hat{g}} + C_g\right)^{-1} \underset{\sim}{g}$$

which is (28).

The associated covariance matrix of $\hat{\underset{\sim}{g}}^*$ is

$$\operatorname{cov}\left(\hat{\underset{\sim}{g}}^*\right) = \left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} \tag{30}$$

This form also can be put in a more tractable form by writing

$$\begin{aligned}
&\left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} \\
&= \left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} I \\
&= \left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1}\left[\left(C_g^{-1} + C_{\hat{g}}^{-1}\right) C_{\hat{g}} - C_g^{-1}\, C_{\hat{g}}\right] \\
&= C_{\hat{g}} - \left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} C_g^{-1}\, C_{\hat{g}} \\
&= C_{\hat{g}} - \left(C_g\left(C_g^{-1} + C_{\hat{g}}^{-1}\right)\right)^{-1} C_{\hat{g}} \\
&= C_{\hat{g}} - \left(I + C_g C_{\hat{g}}^{-1}\right)^{-1} C_{\hat{g}} \\
&= C_{\hat{g}} - \left(C_{\hat{g}}\, C_{\hat{g}}^{-1} + C_g\, C_{\hat{g}}^{-1}\right)^{-1} C_{\hat{g}} \\
&= C_{\hat{g}} - \left(\left(C_{\hat{g}} + C_g\right) C_{\hat{g}}^{-1}\right)^{-1} C_{\hat{g}} \\
&= C_{\hat{g}} - C_{\hat{g}}\left(C_{\hat{g}} + C_g\right)^{-1} C_{\hat{g}}
\end{aligned} \tag{31}$$

As C_g and $C_{\hat{g}}$ play a symetrical role in (30), we have also

$$\left(C_g^{-1} + C_{\hat{g}}^{-1}\right)^{-1} = C_g - C_g\left(C_g + C_{\hat{g}}\right)^{-1} C_g \tag{32}$$

Therefore, when $C_g = 0$ (no errors on data), we have

$$\operatorname{cov}\left(\hat{\underset{\sim}{g}}^*\right) = 0 \tag{33}$$

4. Interpolation

One has often the case of "predicting" a value at any given place (not necessarily at the location of a measurement point). It is the so-called interpolation problem. In this case, one has to use a little trick by supposing that at the location of the interpolation point there is a measurement, but that this measurement is not taken into account. Let us clarify this trick with a little mathematics:

Let $\underset{\sim}{g}$ the vector of the measurements and let g_I a scalar dummy value (not to be taken into account) corresponding to the location of the interpolation point.

Let $\underset{\sim}{G} = \left[\begin{array}{c} \underset{\sim}{g} \\ \hline g_I \end{array}\right]$ the appended vector.

The collocation formula corresponding to (28) is then

$$\hat{\underset{\sim}{G}}^* = C_{\hat{G}}\left(C_{\hat{G}} + C_G\right)^{-1} \underset{\sim}{G} \tag{34}$$

It is very easy of not taking the value of g_I into account by writing

$$C_G = \left[\begin{array}{c|c} C_g & 0 \\ \hline 0 & \sigma_I^2 \end{array}\right] \qquad \text{with } \sigma_I^2 \to +\infty \tag{35}$$

i.e. that the "weight" associated with g_I is zero. If we write $C_{\hat{G}}$ with the same form as

$$C_{\hat{G}} = \left[\begin{array}{c|c} C_{\hat{g}} & C_{\underset{\sim}{\hat{g}I}} \\ \hline C^T_{\underset{\sim}{\hat{g}I}} & C_I^2 \end{array}\right] \text{ where } C_{\underset{\sim}{\hat{g}I}} \text{ is a vector and } C_I^2 \text{ is a scalar.} \tag{36}$$

$$\text{Then } \left(C_{\hat{G}} + C_G\right)^{-1} = \left[\begin{array}{c|c} C_{\hat{g}} + C_g & C_{\underset{\sim}{\hat{g}I}} \\ \hline C^T_{\underset{\sim}{\hat{g}I}} & C_I^2 + \sigma_I^2 \end{array}\right]^{-1} \quad \text{with } \sigma_I^2 \to +\infty \tag{37}$$

By definition of the inverse of matrix, we must have ($\underset{\sim}{\beta}$ is a vector and γ is a scalar)

$$\left[\begin{array}{c|c} C_{\hat{g}} + C_g & C_{\underset{\sim}{\hat{g}I}} \\ \hline C^T_{\underset{\sim}{\hat{g}I}} & C_I^2 + \sigma_I^2 \end{array}\right] \left[\begin{array}{c|c} \alpha & \underset{\sim}{\beta} \\ \hline \underset{\sim}{\beta}^T & \gamma \end{array}\right] = \left[\begin{array}{c|c} I & \underset{\sim}{0} \\ \hline \underset{\sim}{0}^T & 1 \end{array}\right] \tag{38}$$

So,

$$\left(C_{\hat{G}} + C_G\right)\alpha + C_{\underset{\sim}{\hat{g}I}}\underset{\sim}{\beta}^T = I \tag{39a}$$

$$\left(C_{\hat{G}} + C_G\right)\underset{\sim}{\beta} + C_{\underset{\sim}{\hat{g}I}}\gamma = \underset{\sim}{0} \tag{39b}$$

$$C^T_{\underset{\sim}{\hat{g}I}}\alpha + \left(C_I^2 + \sigma_I^2\right)\underset{\sim}{\beta}^T = \underset{\sim}{0}^T \tag{39c}$$

$$C^T_{\underset{\sim}{\hat{g}I}}\underset{\sim}{\beta} + \left(C_I^2 + \sigma_I^2\right)\gamma = 1 \tag{39d}$$

From Eq. (39b) we have

$$\underset{\sim}{\beta} = -\left(C_{\hat{g}} + C_g\right)^{-1} C_{\underset{\sim}{\hat{g}I}}\gamma \tag{40}$$

and therefore from Eq. (39d)

$$\gamma = -\left(C_I^2 + \sigma_I^2 - C^T_{\underset{\sim}{\hat{g}I}}\left(C_{\hat{g}} + C_g\right)^{-1} C_{\underset{\sim}{\hat{g}I}}\right)^{-1} \tag{41}$$

Then $\gamma \to 0$ and $\underset{\sim}{\beta} \to \underset{\sim}{0}$ as $\sigma_I^2 \to +\infty$ and $\alpha \to \left(C_{\hat{g}} + C_g\right)^{-1}$ (42)

At the limit, when $\sigma_I^2 \to +\infty$, Eq. (34) can be written as

$$\underset{\sim}{\hat{G}}^* = \left[\begin{array}{c} \underset{\sim}{\hat{g}}^* \\ \hline \hat{g}_I^* \end{array}\right] = \left[\begin{array}{c|c} C_{\hat{g}} & C_{\underset{\sim}{\hat{g}I}} \\ \hline C^T_{\underset{\sim}{\hat{g}I}} & C_I^2 \end{array}\right] \left[\begin{array}{c|c} \left(C_{\hat{g}} + C_g\right)^{-1} & \underset{\sim}{0} \\ \hline \underset{\sim}{0}^T & 0 \end{array}\right] \left[\begin{array}{c} \underset{\sim}{g} \\ \hline g_I \end{array}\right] \tag{43}$$

$$\Leftrightarrow \left[\begin{array}{c} \underset{\sim}{\hat{g}}^* \\ \hline \hat{g}_I^* \end{array}\right] = \left[\begin{array}{c|c} C_{\hat{g}}\left(C_{\hat{g}} + C_g\right)^{-1} & \underset{\sim}{0} \\ \hline C^T_{\underset{\sim}{\hat{g}I}}\left(C_{\hat{g}} + C_g\right)^{-1} & 0 \end{array}\right] \left[\begin{array}{c} \underset{\sim}{g} \\ \hline g_I \end{array}\right] \tag{44}$$

$$\Leftrightarrow \hat{g}^* = C_{\hat{g}}\left(C_{\hat{g}} + C_g\right)^{-1}\underset{\sim}{g} \tag{45}$$

$$\text{and } g_I^* = C^T_{\underset{\sim}{\hat{g}I}}\left(C_{\hat{g}} + C_g\right)^{-1}\underset{\sim}{g}. \tag{46}$$

Eq (46) is known in the litterature as the "collocation formula".
The associated covariance matrix of $\left(\hat{G}^*\right)$, $\text{cov}\left(\hat{G}^*\right)$ is of course

$$\operatorname{cov}\left(\hat{G}^{*}\right) = \left(C_{\hat{G}}^{-1} + C_{G}^{-1}\right)^{-1} \tag{47}$$

$$= C_{\hat{G}} - C_{\hat{G}}\left(C_{\hat{G}} + C_{G}\right)^{-1} C_{\hat{G}} \tag{48}$$

from Eq. (32).

We have, in block format

$$\operatorname{cov}\left(\hat{G}^{*}\right) = \left[\begin{array}{c|c} C_{\hat{g}} & C_{\underset{\sim}{\hat{g}}l} \\ \hline C_{\underset{\sim}{\hat{g}}l}^{T} & C_{l}^{2} \end{array}\right] - \left[\begin{array}{c|c} C_{\hat{g}} & C_{\underset{\sim}{\hat{g}}l} \\ \hline C_{\underset{\sim}{\hat{g}}l}^{T} & C_{l}^{2} \end{array}\right] \left[\begin{array}{c|c} \left(C_{\hat{g}} + C_{g}\right)^{-1} & \underset{\sim}{0} \\ \hline \underset{\sim}{0}^{T} & 0 \end{array}\right] \left[\begin{array}{c|c} C_{\hat{g}} & C_{\underset{\sim}{\hat{g}}l} \\ \hline C_{\underset{\sim}{\hat{g}}l}^{T} & C_{l}^{2} \end{array}\right] \tag{49}$$

where the inverse of $\left(C_{\hat{G}} + C_{G}\right)$ is built in a manner similar of Eqs. (37) to (41).

We therefore obtain:

$$\operatorname{cov}\left(\hat{G}^{*}\right) = \operatorname{cov}\left(\begin{array}{c} \hat{\underset{\sim}{g}}^{*} \\ \hline \hat{g}_{l}^{*} \end{array}\right) = \left[\begin{array}{c|c} \operatorname{cov}\left(\hat{\underset{\sim}{g}}^{*}\right) & \operatorname{cov}\left(\hat{\underset{\sim}{g}}^{*}, \hat{g}_{l}^{*}\right) \\ \hline \operatorname{cov}^{T}\left(\hat{\underset{\sim}{g}}^{*}, \hat{g}_{l}^{*}\right) & \operatorname{cov}\left(\hat{g}_{l}^{*}\right) \end{array}\right] \tag{50}$$

$$= \left[\begin{array}{c|c} C_{\hat{g}} - C_{\hat{g}}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\hat{g}} & C_{\underset{\sim}{\hat{g}}l} - C_{\hat{g}}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\underset{\sim}{\hat{g}}l} \\ \hline C_{\underset{\sim}{\hat{g}}l}^{T} - C_{\underset{\sim}{\hat{g}}l}^{T}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\hat{g}} & C_{l}^{2} - C_{\underset{\sim}{\hat{g}}l}^{T}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\underset{\sim}{\hat{g}}l} \end{array}\right] \tag{51}$$

i.e.

$$\operatorname{cov}\left(\hat{\underset{\sim}{g}}^{*}\right) = C_{\hat{g}} - C_{\hat{g}}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\hat{g}} \tag{52}$$

$$\operatorname{cov}\left(\hat{g}_{l}^{*}\right) = C_{l}^{2} - C_{\underset{\sim}{\hat{g}}l}^{T}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\underset{\sim}{\hat{g}}l} \tag{53}$$

In fine, the two basic equations of "collocation" are

$$\boxed{\hat{g}_{\underset{\sim}{l}}^{*} = C_{\underset{\sim}{\hat{g}}l}^{T}\left(C_{\hat{g}} + C_{g}\right)^{-1} \underset{\sim}{g}} \tag{54}$$

$$\boxed{\operatorname{cov}\left(\hat{g}_{l}^{*}\right) = C_{l}^{2} - C_{\underset{\sim}{\hat{g}}l}^{T}\left(C_{\hat{g}} + C_{g}\right)^{-1} C_{\underset{\sim}{\hat{g}}l}} \tag{55}$$

ANNEXES

1. Least squares estimation

In basic least-squares, one wants to "solve" the non-square (usually overdetermined) system

$$A\,\underset{\sim}{x} = \underset{\sim}{b} \qquad \text{(A-1)}$$

As A^{-1} does not exist, one seeks to minimize the distance

$$Q = \left\| A\,\underset{\sim}{x} = \underset{\sim}{b} \right\| \qquad \text{(A-2)}$$

w.r.t. some suitable norm.
The least squares norm is defined by

$$Q = \left(A\,\underset{\sim}{x} = \underset{\sim}{b}\right)^T P \left(A\,\underset{\sim}{x} = \underset{\sim}{b}\right)$$

where P is a real symetric definite positive matrix, called "weight matrix", such as Q is a strictly positive quantity.
The minimum of Q is given by

$$\delta Q = 0 \quad \text{w.r.t. } \underset{\sim}{x} \qquad \text{(A-3)}$$

i.e.

$$\delta\left[\left(A\,\underset{\sim}{x} = \underset{\sim}{b}\right)^T P \left(A\,\underset{\sim}{x} = \underset{\sim}{b}\right)\right] = 0$$

$$\Leftrightarrow \delta\left[\left(\underset{\sim}{x}^T A^T - \underset{\sim}{b}^T\right) P \left(A\,\underset{\sim}{x} - \underset{\sim}{b}\right)\right] = 0$$

$$\Rightarrow \delta\left(\underset{\sim}{x}^T A^T A\,\underset{\sim}{x} - 2\,\underset{\sim}{b}^T P A\,\underset{\sim}{x}\right) = 0$$

$$\Leftrightarrow \left(2\,\underset{\sim}{x}^T A^T P A - 2\,\underset{\sim}{b}^T P A\right)\delta\,\underset{\sim}{x} = 0$$

$$\Rightarrow \underset{\sim}{x}^T A^T P A - \underset{\sim}{b}^T P A = 0$$

$$\Leftrightarrow A^T P A\,\underset{\sim}{x} = A^T P\,\underset{\sim}{b} \qquad \text{(A-4)}$$

$$\Rightarrow \underset{\sim}{x}^* = \left(A^T P A\right)^{-1} A^T P\,\underset{\sim}{b} \quad \text{if } \left(A^T P A\right)^{-1} \text{ exists.} \qquad \text{(A-5)}$$

Let us note that this definition of the least squares solution is purely algebric.

2. Errors on the second member $\underset{\sim}{b}$

In the real world, the vector $\underset{\sim}{b}$ is affected by measurements errors. How these errors map in the estimated vector $\underset{\sim}{x}^*$?

We have first to find a way to characterize error in $\underset{\sim}{b}$, or in any vector $\underset{\sim}{u}$.

By definition, if we have a set $\left\{\underset{\sim 1}{u}, \underset{\sim 2}{u}, \ldots, \underset{\sim N}{u}\right\}$ representing measurements of a vector $\underset{\sim}{u}$ (for example repeated surveys over the same set of gravimetric stations), we define the mean vector $\underset{\sim o}{u}$ of $\underset{\sim}{u}$ as :

$$\underset{\sim o}{u} = M\left(\underset{\sim}{u}\right) = \frac{1}{N}\sum_{i=1}^{N} \underset{\sim i}{u} \tag{A-6}$$

and the covariance of $\underset{\sim}{u}$ as the matrix of the deviations of the components of $\underset{\sim}{u}$ with respect to their means

$$\begin{aligned} \operatorname{cov}\left(\underset{\sim}{u}\right) &= M\left[\left(\underset{\sim}{u} - \underset{\sim o}{u}\right)\left(\underset{\sim}{u} - \underset{\sim o}{u}\right)^T\right] \\ &= \frac{1}{N}\sum_{i=1}^{N}\left(\underset{\sim i}{u} - \underset{\sim o}{u}\right)\left(\underset{\sim i}{u} - \underset{\sim o}{u}\right)^T \end{aligned} \tag{A-7}$$

Then we have, for the mean value $\underset{\sim o}{x^*}$ of $\underset{\sim}{x}$

$$\begin{aligned} \underset{\sim o}{x^*} &= M\left(\underset{\sim i}{x^*}\right) \\ &= M\left(\left(A^T P A\right)^{-1} A^T P \underset{\sim}{b}\right) \\ &= \frac{1}{N}\sum_{i=1}^{N}\left(A^T P A\right)^{-1} A^T P \underset{\sim i}{b} \\ &= \left(A^T P A\right)^{-1} A^T P \frac{1}{N}\sum_{i=1}^{N} \underset{\sim i}{b} \\ &= \left(A^T P A\right)^{-1} A^T P \underset{\sim o}{b} \end{aligned} \tag{A-8}$$

and for the covariance $\operatorname{cov}\left(\underset{\sim}{x^*}\right)$ of $\underset{\sim}{x}$

$$\begin{aligned} \operatorname{cov}\left(\underset{\sim}{x}\right) &= M\left(\left(\underset{\sim}{x} - \underset{\sim o}{x^*}\right)\left(\underset{\sim}{x} - \underset{\sim o}{x^*}\right)^T\right) \\ &= M\left(\left(A^T P A\right)^{-1} A^T P\left(\underset{\sim}{b} - \underset{\sim o}{b}\right)\left(\underset{\sim}{b} - \underset{\sim o}{b}\right)^T P A\left(A^T P A\right)^{-1}\right) \\ &= \left(A^T P A\right)^{-1} A^T P\, M\left[\left(\underset{\sim}{b} - \underset{\sim o}{b}\right)\left(\underset{\sim}{b} - \underset{\sim o}{b}\right)^T\right] P A\left(A^T P A\right)^{-1} \\ &= \left(A^T P A\right)^{-1} A^T P \operatorname{cov}\left(\underset{\sim}{b}\right) P A\left(A^T P A\right)^{-1} \end{aligned} \tag{A-9}$$

By defining now P as

$$P = \left[\operatorname{cov}\left(\underset{\sim}{b}\right)\right]^{-1} = C_b^{-1} \tag{A-10}$$

we have

$$\begin{aligned} \operatorname{cov}\left(\underset{\sim}{x}\right) &= \left(A^T C_b^{-1} A\right)^{-1} A^T C_b^{-1} C_b C_b^{-1} A\left(A^T C_b^{-1} A\right)^{-1} \\ &= \left(A^T C_b^{-1} A\right)^{-1} \end{aligned} \tag{A-11}$$

which is Eq. (4).